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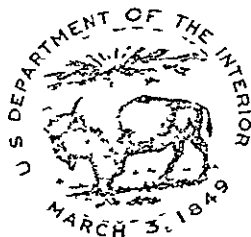
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GRAVITY ANALYSIS



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COMBINED MAGNETIC AND
GRAVITY ANALYSIS

A study of the feasibility of enhancing
the geologic interpretation of satellite
magnetic data by combined magnetic and gravity analysis.

Final Report, NASA Contract No. S-500 29A Modification No. 8

U.S. Geological Survey

by

W. J. Hinze*†, L. W. Braile†,

V. W. Chandler*†, F. E. Mazzella†

†Department of Geosciences, Purdue University, West Lafayette, Indiana

*also W.A.E., U.S. Geological Survey, Reston, Virginia

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I. INTRODUCTION

Magnetic methods have been used for centuries to investigate the earth's crust. Over the years it has been the backbone of geophysical exploration in the mineral industry. More recently it has become an important reconnaissance technique in petroleum exploration. During the past few decades magnetics has had an increasingly significant role in regional crustal studies and in fact has triggered today's revolution in the geosciences associated with the development of the concepts of plate tectonics and sea floor spreading. Looking to the future, additional applications of the magnetic method are anticipated in regional crustal problems as magnetic coverage of the earth increases by employing magnetic measuring satellites. The impending availability of widespread magnetic data has focused attention on improved interpretational techniques.

Geologic interpretation of magnetic anomalies suffers from a high degree of ambiguity. This ambiguity is inherent to the interpretation of all potential fields and thus plaques the interpretation of gravity anomalies as well. An extension of the Green's Theorem of Equivalent Layer shows that observed anomaly values can be reproduced by an infinite number of surface distributions shallower than the maximum possible source of the anomaly. This lack of uniqueness in the interpretation cannot be eliminated by measuring gradients or anomalies at various elevations because these are not independent parameters. Additional ambiguity in anomaly interpretation is derived from superposition of anomalies and inadequate isolation of anomalies. Furthermore, magnetic interpretation

is impaired by the effects of remanent magnetization which is superimposed on the magnetization induced in rocks by the earth's magnetic field.

The ambiguity of magnetic interpretation can be decreased with constraints placed upon the interpretation by direct geologic information and by extrapolating from known geology to the unknown with magnetic data. However, these approaches are limited to areas where the magnetic rocks outcrop or are encountered in drilling. Another approach to the solution of this problem is to combine the interpretation of magnetic anomalies with gravity anomalies assuming anomalies are derived from a common source. Gravity and magnetic anomalies are commonly derived from a singular source, but of course this is not a universal situation. Present techniques of combining the analysis of gravity and magnetic anomalies are largely restricted to visual spatial correspondence of anomalies on either maps or profiles and independent source parameter interpretations from each anomaly and subsequent synthesis and correlation of interpretations. These interpretations may be iterated to increase the correspondence of the calculated source parameters from individual anomalies.

Preliminary studies indicate that other methods of interpretation based upon a less subjective and thus more quantitative approach are possible and highly desirable to improve our knowledge of the geology and geophysical properties of the lithosphere. The objective of this study is to identify methods of decreasing magnetic interpretation ambiguity by combined gravity and magnetic analysis, to evaluate these techniques in a preliminary manner, to consider the geologic and geophysical

implications of correlation, and to recommend a course of action to evaluate methods of correlating gravity and magnetic anomalies. The particular emphasis of this study is toward the interpretation of magnetic data collected at satellite elevations, but the techniques considered have broad application to the geophysical sciences in the interpretation of gravity and magnetic data for geologic, petroleum, and mineral exploration purposes.

The major thrust of the study to achieve the stated objectives was a search and review of the literature. The literature of geophysics, geology, geography, and statistics was searched for articles dealing with spatial correlation of independent variables. Emphasis was placed on the correlation of gravity and magnetic anomalies, but was not limited to these variables. An annotated bibliography referencing the germane articles and books is presented. In the second chapter the methods of combined gravity and magnetic analysis techniques are identified and reviewed. The third and fourth chapters are concerned with a more comprehensive evaluation of two types of techniques. The third deals with internal correspondence of anomaly amplitudes, which is a zero lag cross-correlation scheme using a limited-size moving data window, and clustering and characterization techniques. These are investigated utilizing empirical model studies. The fourth chapter is directed toward combined analysis utilizing Poisson's theorem. The fifth section discusses the geologic and geophysical implications of gravity and magnetic correlation based on both theoretical and empirical relationships.

II. COMBINED MAGNETIC AND GRAVITY

ANALYSIS TECHNIQUES

Introduction

A common approach to magnetic interpretation is to compare magnetic data either in profile or map form to corresponding spatial variations of other geophysical parameters or geologic variables or to magnetic data from other areas. The purpose of these comparisons is to determine similarity between areas, to extrapolate known geologic conditions into unknown areas by magnetic data and to decrease the ambiguity of the magnetic interpretation. The central theme of this discussion is a consideration of techniques to achieve the latter purpose, that is to decrease the ambiguity of magnetic interpretation. The geologic interpretation of magnetic data as explained in the previous chapter is subject to considerable ambiguity due to inherent restrictions in potential theory and problems associated with the geologic and geophysical characteristics of the geologic sources.

One method of enhancing magnetic interpretation is to perform combined magnetic and gravity analysis. Numerous magnetic and gravity surveys of the same area and theoretical considerations show that variations in density which produce gravity anomalies are commonly related to magnetization variations which cause magnetic anomalies. This is particularly true of gravity anomaly sources occurring in igneous and metamorphic rocks because of the generally low values of magnetization of sediments and sedimentary rocks. Even the lack of a relationship between magnetic and gravity anomalies can be informative about the geology of

an area by using basic geologic and geophysical concepts. Another reason for considering combined magnetic and gravity analysis is the increasing availability of world-wide gravity measurements to relate to magnetic measurements made from airborne platforms, particularly satellite magnetic data.

Combined magnetic and gravity analysis techniques cover a broad range of methodologies to determine the degree and direction of correspondence and relationship between these two independently measured potential fields. Correlation is the general term used in this discussion for these techniques, but it is used in a far broader sense than the simple statistical definition of correlation. Thus, "correlation" is used as a broad umbrella term to cover "qualitative", "semi-quantitative", and "quantitative" methods of combined magnetic and gravity analysis. These terms are enclosed in quotation marks to emphasize that they are relative terms. A flow chart of combined magnetic and gravity analysis (Figure 1) separates the various techniques under these headings on the basis of their approach and the degree of subjectivity involved in their interpretation. "Qualitative" methods have the highest degree of subjectivity and "quantitative" methods have the lowest. For example, "qualitative" methods may simply involve an overlay of gravity and magnetic anomaly maps to visually determine their degree of similarity, while "quantitative" techniques may use Poisson's theorem to determine the direction of magnetization within a causative geologic body.

The following discussion of combined magnetic and gravity analysis follows the flow chart shown in Figure 1. In general, magnetic and gravity

data, in map or profile form and either in analog or digital format, may be subjected to a variety of pre-processing steps to prepare the data for correlation. In addition, for some types of correlation procedures the magnetic and gravity data must be registered by obtaining digital data at common points. The data may then be correlated by one or more of five basic techniques which fall under the "qualitative", "semi-quantitative", or "quantitative" groupings.

The correlation techniques lead to a number of possible results that are shown in rectangles on the flow chart. The ultimate result is of course the geologic interpretation which is shown at the bottom of the flow chart.

For ease in relating the combined magnetic and gravity analysis techniques to the references given in the annotated bibliography (Appendix) a flow chart (Figure 2) has been prepared which refers the particular method to appropriate numbered references, the annotated bibliography is limited to references that were available in english for review and contains only representative articles dealing with qualitative correlation.

Pre-Processing

Magnetic and gravity data may be subjected to one or more pre-processing steps to facilitate the correlation procedure. A wide variety of techniques of achieving these techniques have been discussed in the geophysical literature. Therefore, the procedures will not be discussed in detail here. Fourier transforms have been used in combining gravity and magnetic anomaly data with Poisson's theorem by Kanasewich and Agarwal (1970) and Cordell and Taylor (1971) and Bhattacharyya (1965) and others

have used Fourier transforms to calculate the magnetic field reduced to the pole, upward continuation and a host of other potential field qualities. Baranov (1957), Baranov and Naudy (1964), Bhattacharyya (1965) and others have discussed methods of transforming the magnetic field to the pole to eliminate the distorted magnetic anomaly pattern produced by non-vertical magnetization. Application of this technique to magnetic data, particularly data observed at low magnetic latitudes will aid in visual and analytical spatial correlation. Wavelength filtering of magnetic and gravity data as suggested by Dean (1958), Robinson (1970) and others may be used to isolate particular anomalies for correlation. Upward continuation as suggested by Peters (1949) and others may be used to smooth gravity and magnetic data for correlation and to place surface gravity anomalies at the same elevation as airborne magnetic observations for processing by Poisson's theorem and other correlation techniques.

"Qualitative" Correlation

"Qualitative" correlation as used in the combined magnetic and gravity analysis flow chart involves two basic approaches to correlation, visual spatial correlation and inverse interpretation. Currently, of all the combined analysis techniques, the most widely used correlation scheme is visual spatial correlation. This involves a technique which has long been used by geoscientists whereby a subjective, qualitative correlation is made by the overlay of maps and profiles. Correlation is used here in its broadest sense and not in a strict statistical definition. In visual correlation the analyst searches for a spatial coincidence of

gravity and magnetic anomalies. A coincidence of anomalies suggests a common source and the relative amplitudes and their sign, gradients, shape, strike and other parameters which can be visually characterized are used together with geologic concepts and a knowledge of rock properties to arrive at a geologic interpretation. The results are largely subjective and the accuracy of the interpretation is strongly biased by the experience of the interpreter and the geological background and concepts used in the analysis.

Visual spatial correlation has been primarily used in the analysis of basement rocks because the igneous and metamorphic rocks which make up the basement commonly show both magnetization and density variations. Furthermore, there is a likelihood of a general correspondence between densities and magnetizations (Nettleton and Elkins, 1944; Garland, 1951). However, there are many exceptions to this correspondence and Affleck (1957) considers that magnetizations are much less uniform than densities. This is also shown to be true by the multitude of magnetic susceptibility and density (specific gravity) measurements made by Werner (1945) on acidic and basic rocks, sedimentary rocks and iron ores (Figure 3). The densities of acidic and basic rocks vary by much less than an order of magnitude, while susceptibilities vary by over four orders of magnitude. Undoubtedly, this is at least in part due to the fact that magnetic susceptibility is caused primarily by a minor mineral (magnetite) which does not affect the density appreciably. Other reasons for the lack of correlation between gravity and magnetic anomalies include remanent magnetization effects and the variable effect of depth which

causes the amplitude of magnetic anomalies to decrease one power faster than gravity anomalies from the same source. Chereau and Naudy (1967) discuss other reasons for lack of correlation. Nonetheless, visual spatial correlation has been useful in basement geology analysis. Representative examples are given by Woollard (1943 and 1959), Henderson and Zietz (1958), Leney (1966), Chereau and Naudy (1967), MacLaren and Charbonneau (1968), Hinze and Merritt (1969), Lidiak (1971), King and Zietz (1971), Rudman and others (1972), and Eaton and others (1975).

Visual spatial correlation can be enhanced by removing the effect of horizontal magnetization by transforming the magnetic anomaly to the pole and comparing this field with the vertical gradient of gravity. The vertical gradient of gravity is related to the magnetic field at the pole through Poisson's theorem by a constant which includes the ratio of the magnetization to density. This technique has been used by Chereau and Naudy (1967).

The other basic approach to qualitative correlation, inverse interpretation, involves independent source parameter interpretation from the gravity and magnetic anomalies using standard modeling procedures and subsequent synthesis and correlation of interpretations. These interpretations by source modeling may be iterated to increase the correspondence of the calculated source parameters from the individual anomalies. The source parameters are then used to derive a geologic interpretation. The calculated source parameters and geologic interpretation are not necessarily unique, but greater confidence can be placed in the combined interpretation than in the inverse interpretation of a single force field

anomaly. This approach to combined gravity and magnetic analysis has been used by several investigators. Typical examples have been given by Hinze and Merritt (1969), Rudman and Blakely (1965), and Oray and others (1973).

"Semi-Quantitative" Correlation

Semi-quantitative correlation techniques which provide combined magnetic and gravity analysis by a variety of statistical methods have been used to only a minor degree in geophysical interpretation. As shown in Figure 1 there are two broad general classes of these methods, clustering and characterization and analytical spatial correlation.

A search of the literature has found no example of the use of clustering and characterization in combined magnetic and gravity analysis, however, it has been emphasized in magnetic interpretation using a limited number of parameters (Hall, 1964).

Clustering and characterization refers to statistical correlation of multi-parameter data to define point or areal data that have common characteristics within certain limits. Its purpose here is to classify geographical areas into more or less homogeneous groups so that areas of similar geophysical parameters can be identified and mapped. Geophysical analysts are very well acquainted with delineating geologic zones on the basis of similarity in the "character" of anomalies. This is commonly a step in visual spatial correlation described under qualitative interpretation. Clustering and characterization is designed to minimize the subjectivity of this approach and to make it possible to handle more parameters

than can usually be considered in visual techniques. Magnetic and gravity data generally consist of the measurement of a single parameter, the amplitude of some component of their respective force fields. However, a host of parameters describing and derived from the inter-relationship of neighboring data points are available to the analyst. The sum total of these parameters define the "character" of the anomalies. Affleck (1963) has discussed a number of these features or parameters found on magnetic maps. Similar parameters are available from gravity data and still more are defined by the correlation of magnetic and gravity parameters. The use of both gravity and magnetic data in the classification of areas should enhance the discrimination procedure.

A variety of multi-parameter analysis procedures have been developed (Davis, 1973). They are complicated in their theoretical structure and operational methodology and as pointed out by Davis "For many of the procedures, statistical theory and tests have been worked out only for the most restricted set of assumptions." However, the general procedure as illustrated in Figure 1 involves determination of the critical parameters of the gravity and magnetic data, correlation of the selected critical parameters, and identification of the classifiers from the correlation procedure. These classifiers are then used to isolate geologic zones of homogeneous source parameters.

Simplified, preliminary applications of this technique are discussed and illustrated in the next chapter.

Analytical spatial correlation, the second general group of methods in semi-quantitative correlation, is concerned with the quantitative

comparison of gravity and magnetic anomaly maps or portions of these maps. The quantitative comparison of maps has been a subject of considerable interest to geographers and geologists, but has received only limited attention from geophysicists. Three general methods have been suggested; internal correspondence, cross-correlation, and surface coefficient correlation.

Internal correspondence is a procedure defined by Robinson (1962) to determine the spatial similarity between maps of different parameters of the same area by zero-lag cross-correlation using a limited size moving data window. The same procedure can be used on profiles. Empirical studies reported on in the next chapter suggest that in addition to the zero-lag cross-correlation value or the coefficient of correlation, critical information can be derived from the slope and intercept of the least squares line fitted to the data within the window. The application of this technique to theoretical and observed data is illustrated and discussed in the next chapter. It is apparent that this technique holds considerable potential in combined magnetic and gravity analysis, but many questions remain to be explored. These include the effect of window size, removal of trends or regionals, methods of interpreting the data, effect of normalizing and standardizing data and others.

Botezatu and Calota (1973) have studied the properties of non-normalized cross-correlation functions of gravity and magnetic anomalies derived from idealized sources. They show that the function can be used to discriminate between genetically related anomalies and separate sources situated on a vertical line. They have applied their method to force

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field data from Romania with success.

The majority of the work done in analytical spatial correlation has been done through variations of the surface coefficient correlation technique, although the literature on the subject is not extensive. The general procedure of this technique is to compare the mathematical expressions of the surfaces. Hide and Malin (1970) have correlated and tested the correlation of selected coefficients of the spherical harmonic expansions of the geomagnetic and the earth's gravity field. A similar approach has been used by Merriman and Sneath (1966). They compare the coefficients of well-fitting surfaces of the same order. According to Bassett's (1972) review, "if orthogonal polynomials are used to fit surfaces to regularly spaced data the successive coefficients are independent. Each coefficient number can be regarded as an orthogonal dimension and each surface can be represented as a point in the resulting multidimensional space. A variety of distance grouping procedures is then appropriate." Mandelbaum (1966) has pointed out some potential practical limitations to this approach. Merriam and Lippert (1966) compared residuals from trend surfaces and calculated the coefficients of association based on the number of matches between residual maps. Curry (1967) has suggested fitting a polynomial to one surface and then reducing a second surface by the same expression. "The measure of association would be the proportion of the variance of the second map explained by the polynomial of the first."

Both Bassett (1972) and Davis (1973) have more detailed reviews of

surface coefficient correlation techniques. The application of these methods to regional magnetic and gravity anomaly maps remains untested. Their effectiveness in dealing with magnetic maps which contain relatively high frequency components is in doubt. However, these techniques may be applicable to satellite magnetic observations which are devoid of strong high frequency anomaly components. Further testing and evaluation of these techniques are definitely warranted.

"Quantitative" Correlation

Poisson in 1826 discussed the mathematical relationship between magnetic and gravitational potentials associated with any body that is homogeneously magnetized and dense. These potentials are related to force fields at any position in a manner that their derivatives or gradients in a direction equals the magnitude of the force in that direction. Thus Poisson's theorem

$$V = \frac{J}{G\sigma} \frac{\partial U}{\partial i}$$

where V is the magnetic potential of a source

U is the gravitational potential of a source

J is the magnetization contrast of the source with the surrounding rocks

σ is the density contrast of the source with the surrounding rocks

i is the direction of magnetization of the source

G is the gravitational constant

can be used to relate gravity anomalies ($\partial U / \partial z$) with magnetic anomalies (e.g., vertical magnetic anomalies, $\partial V / \partial z$). Utilizing Poisson's theorem and observed gravity and magnetic anomalies, it is possible to determine

characteristics of the source in a much more definitive manner than from the interpretation of a single force field anomaly. Thus correlation techniques employing numerical application of Poisson's theorem are referred to as "quantitative" in Figure 1. A complete list of references annotated in the appendix on the theory and application of Poisson's theorem is given in Figure 2.

Despite the great potential of Poisson's theorem in quantitative combined magnetic and gravity analysis only approximately a dozen references have been found which discuss applications of this method. This undoubtedly is due in large part to the assumptions that are necessary to implement it. The assumptions can never be met, but only approached in practical cases. Thus there is a strong need to relate the accuracy of the assumptions to the correctness of the results obtainable by employing Poisson's theorem. Until this is achieved, Poisson's theorem will remain a mathematically interesting technique which is only used under specialized circumstances. Further discussion of Poisson's theorem, its application and limitations, is developed in Chapter IV.

III. COMBINED ANALYSIS USING INTERNAL CORRESPONDENCE AND CLUSTERING

Introduction

This chapter treats in a preliminary way two potentially important techniques of combined magnetic and gravity analysis, internal correspondence and cluster analysis. Internal correspondence is based on a procedure first outlined by Robinson (1962) to deal with geographic data. In this method, gravity and magnetic maps or profiles are divided into equal segments and a least squares linear regression between the gravity and magnetic anomaly amplitudes is conducted within each segment. The spatial variation of the regression coefficients are used to analyze the relationships between the profiles or maps. Several problems involved in the application of this method, such as data standardization, segment (or window) size, the effect of body depths, and the meaning of the regression coefficient values, will be discussed. Two model and one observed data profile will also be analyzed.

Cluster analysis is a general term including several rather complex statistical techniques. However, the objective of each technique is basically the same; classifying similar objects into common groups based on variables found in each object. Little has been done in the application of this procedure to geophysics, but there is no reason why gravity and magnetic maps or profiles cannot be divided into smaller segments and treated as objects in a clustering procedure. A variety of variables describing aspects of the gravity and magnetic data are available to the geophysical analyst. The clustering algorithm used in this study is from Davis (1973). Clustering as applied to gravity and magnetic data is yet in its infancy, but preliminary results indicate that it may prove to be

of great value in mapping regional geology.

Internal Correspondence Analysis

Method

One method commonly used by geographers when comparing two contour maps involves linear regression and the generation of so-called residual maps. In this technique one set of data is selected as the independent variable and a linear regression is made over the entire data set. From this, a map of the regressed dependent variable is constructed and subtracted from the observed dependent variable map. The residual map, the regressed map, and the regression coefficients are then used to determine the relationship between the two original maps.

This approach, however, is of limited use in the analysis of gravity and magnetic data. Over a large area, the relationship between gravity and magnetic anomalies change and thus a linear relationship is not applicable. Regressed lines were fitted to the scatter diagrams of Profiles 1 and 2 (Figures 9 and 12) and it is clear that a linear fit oversimplifies the actual relationship between the gravity and magnetic data.

An alternative to whole map correlation has been suggested by Robinson (1962) in a study of the relationship between two sets of contoured geographic data. The procedure consists of first dividing the regional area into a number of smaller square subareas. Within each subarea, the correlation coefficient between the two sets of data is calculated and plotted at its center. Finally a correlation coefficient contour map is plotted and analyzed. Robinson calls this

method "internal correspondence" and it would seem that it can be applied directly to gravity and magnetic contour maps.

Nevertheless, several improvements should be applied to Robinson's procedure to strengthen its use in the analysis of gravity and magnetic data. First, the choice of proper subarea (or window) size for a particular set of data is still a rather arbitrary procedure (Robinson, personal communication) and some improvement on this matter may be helpful. Second, Robinson's technique only includes the correlation coefficient which carries with it no information regarding the relative magnitudes of variation between the two data sets. Thus, for example, a correlation coefficient cannot differentiate between a comparison of a small gravity anomaly to a large magnetic anomaly or vice versa; it merely gives the strength of the linear relationship between the two data sets within a given subarea. The regression coefficients, especially the so called slope coefficient, may yield information regarding the relative magnitudes of variation between the gravity and the magnetic data within a given subarea. Thus, linear regression coefficients will be used in the internal correspondence analysis.

The data analyzed by internal correspondence in this report are all in the form of profiles. Thus it is assumed that the gravity and magnetic data are two-dimensional or strike infinite perpendicular to the profile. A subarea is simply a segment of the profile. A vertical magnetic field of 58,000 gammas and a common level of observation for the gravity and magnetics is also assumed for all profiles. The two models are 400 km in length and are sampled at an interval of 0.5 km.

Before analysis is made of the gravity and magnetic data, the data are standardized. Every data point has the mean of the entire population (here the profile) subtracted from it and the remainder is divided by the standard deviation of the population. The resulting number is what will be used in the internal correspondence. Standardization is generally recommended when two sets of variables having different units of measurement are being regressed. The use of standardized data will also assist in the interpretation of internal correspondence.

A computer program was designed at Purdue University to do the actual calculations involved in the internal correspondence analysis. First, the digitized values along the gravity and magnetic profiles are read in, as well as the size of the window or subarea to be analyzed. After standardization, the window is centered over the first data point and a least squares linear regression is performed over the data within the window. The resulting slope, intercept, and correlation coefficients are stored into arrays, the window is shifted one position over, and the process is repeated. When completed, every point along the profile will have a corresponding slope, intercept, and correlation coefficient value. These values are then plotted as three profiles for visual inspection.

The problem of selection of optimum window size depends primarily on the width of the anomalies to be correlated. During this study it became apparent that the wider windows are associated with lower correlation coefficients, especially if the region is characterized by relatively narrow anomalies. It is doubtful that any coefficients would be significant if they are derived from a window producing an absolute correlation value below 0.5. Thus, the value of the correlation coefficient can be helpful in selecting an upper bound for window size. The lower bound for

window size is related to the minimum number of points required for an accurate regression analysis and the narrowest anomalies of interest in the area. This problem of window size will be further pursued in the analysis of Profile 1.

Significance of Internal Correspondence

Before going into the analysis of several theoretical and observed data profiles, it is appropriate to outline some of the basic concepts of this relatively unexplored technique in terms of the three coefficients and the scatter diagrams.

The correlation coefficient defines how well a change in gravity is reflected by a linear change in magnetics within a given window. Inverse relationships are given by negative coefficients. However, as stated before, the correlation coefficient is devoid of information regarding relative magnitudes of change. As the geologic interpretation of an area depends heavily on the magnitudes of the gravity and magnetic anomalies it is necessary to use an additional parameter.

Regression coefficients, especially the slope coefficient, have proven useful in expressing the magnitude relationships between gravity and magnetic anomalies. For the sake of consistency, gravity will always be regressed to the magnetics. Thus, the regression within each window will be of the form:

$$m = gS + I$$

where m is the magnetic value estimated by regression, g is the gravity value, S is the slope coefficient of the regression, and I is the intercept of the regressed line onto the magnetic axis.

The value of the slope coefficient from a regression using standardized data strongly reflects the effect of the relative magnitudes between a gravity and magnetic anomaly within a given subarea. If the slope value within a window is one, the variation of the standardized gravity is equal to the variation of the standardized magnetics. For example, a slope coefficient value of one would occur when a subarea has a large gravity anomaly matched by a large magnetic anomaly.

A slope value in excess of one means that a small variation in gravity within a window is matched by a large change in magnetics. The limit occurs as the slope goes to infinity meaning that a variation in the magnetics has no corresponding variation in the gravity. In contrast, a slope value less than one indicates that a large variation in gravity is matched by only a small variation in magnetics. Thus, two unmatched amplitude relationships can exist between gravity and magnetic data, one as the slope value tends to infinity and one as the slope value tends to zero.

The slope parameter takes on the same signs as the correlation coefficient plus yields a number expressing the relationship of the magnitudes of variation between the two data sets. Therefore, the slope parameter is critical to internal correspondence analysis. However, a slope value should be cross-checked with its corresponding correlation coefficient to be certain that there is a significant relationship. Should the relationship within a given window fall below an absolute value of correlation of 0.5, the slope value should be regarded with caution.

The intercept coefficient does not have the same sign relationship as the correlation or slope coefficients, thus it does not have the ability to separate direct from inverse relationships between gravity and magnetics. Therefore, at present, the use of the intercept coefficient in internal correspondence is limited. Like the slope parameter, an extremely large value indicates that a low variation in gravity is matched with a large variation in the magnetics.

Included with every internal correspondence analysis is a scatter diagram of the gravity versus magnetic values for every data point along the profile. Two varieties of this diagram are presented. The first diagram shows the gravity and magnetic values along each point in the profile as an asterisk. A least squares line fitted to all the data is also included in this diagram. The second type of diagram shows the same points, but now they are joined in sequence by a line representing their order in the profile. Horizontal distances at every 20 km are plotted along this curve. Internal correspondence can be seen as a performing linear regression within a moving window over the profile as represented on the gravity-magnetic scatter diagram. Though no detailed analysis will be made of these diagrams in this study, they have proven to be a helpful supplement during both the internal correspondence and clustering analysis.

Internal Correspondence Analysis of Profile One

Profile 1 is a model that consists of three sizes of square cross-sectioned two-dimensional bodies arranged at three different depths. There are bodies one by one km on a side at one km depth,

bodies 5 by 5 km at 5 km depth, and bodies 5 by 30 km at a depth of 20 km (Figure 4). The bodies display a variety of density contrasts ranging from -0.3 to $+0.3$ g/cc and magnetic susceptibility contrasts ranging from -6000 to $+6000 \times 10^{-6}$ emu/cc (Table 1). Proceeding from left to right, the shallow bodies are numbered 1 to 10, the intermediate bodies from 11 to 17 and the deep bodies from 18 to 20.

The observed gravity and magnetics of Profile 1 (Figure 5) present anomalies of three different widths. In order to observe the effect of window width, windows equal to 2.5, 12.5, and 22.5 km were selected for the internal correspondence analysis (Figures 6, 7, and 8). The selected window sizes correspond approximately to the three anomaly half-widths at one-half the maximum amplitude.

The region between 0-180 km is characterized by shallow and intermediate bodies that generally display large magnetic anomalies but only small gravity anomalies. This situation gives rise to extreme slope and intercept values as observed in Figure 6. Elsewhere in the profile the shallow and intermediate bodies are less magnetic and their slope and intercept values are correspondingly much lower.

The narrow anomalies are best emphasized at narrower window lengths but the highly magnetic zone from 0-180 km gives rise to relatively high slope and intercept values even at wider window intervals (Figure 8). The effect of the deeper bodies appears to be best observed at wider window intervals. The area between 180 and 270 km shows up as a general region of predominantly negative correlation at a window of 22.5 km (Figure 8) and most certainly is the effect of the high density low susceptibility deep body that underlies this area. The effects of

the large regional scale anomalies can be seen on the internal correspondence results of the small bodies, even at narrow window widths (Figure 6).

The scatter diagrams for this profile yield several interesting patterns which correspond to the various types of bodies involved (Figure 9). The regional anomalies show up as gently sloping to nearly horizontal broad curves on the profile. Regions numbered 200-260 or 300-400 are regions dominated by the effect of deep seated regional anomalies. The shallower, more magnetic bodies show up as nearly vertical, sharp peaks. Their configuration clearly emphasizes the high slope and intercept values observed during the internal correspondence analysis.

Analysis of Profile Two

The second model was designed primarily for cluster analysis, but a brief account of the internal correspondence results is warranted. The model represents an area of four distinct igneous provinces (Figures 4 and 10). The region between 0 and 130 km is characterized by large 5 x 5 km plutons of diorite. From 130-240 km is an area intruded by 1 km thick dikes of basalt, most of which are vertical. Between 320-400 km occur two large triangular masses of granite. Finally, between 240-320 kilometers a mixture occurs of the three previous rock types in a variety of shapes and depths. The density and magnetic susceptibility contrasts for the bodies are given in Table 2.

Internal correspondence of this profile using a window of 7.5 km demonstrates several important aspects of the method (Figure 11).

An increase in depth corresponds to a decrease in slope coefficients over bodies 1 vs. 2 and 6 vs. 7. In addition the shapes of the bodies affect the shape of the slope coefficient profiles. Thus, depths and shape of the body as well as the magnetization/density ratio, affect the value of the slope coefficient. It is also noteworthy that interference between two anomalies can generate slope coefficient values that are unusually high, such as those observed at 90 km and between 240-310 km. It is encouraging that the slope coefficient curves show different values and shapes over different geological sources.'

The scatter diagrams (Figure 12), although complex in pattern, do delineate the three basic lithologies. The curves which occur to the left of the diagrams are associated with the granite bodies. The steeper curves appear to roughly fall into two groups. The group to the right is associated with the basaltic bodies and the one on the left is associated with the diorite plutons. Therefore, internal correspondence results of the model profiles indicate that the coefficients are sensitive to different geologic sources and that there is potential for using internal correspondence to infer geologic parameters.

Analysis of Woollard's Transcontinental Profile

As a further test for the internal correspondence analysis, Woollard's (1943) transcontinental profile of North America was selected. The choice of this survey over several more recent works of a similar nature was based on two reasons. First, both the gravity and magnetic data were taken at the same level. Thus, no upward continuation was necessary to

bring the gravity and magnetic data to a common level of observation. Woollard's observations are of the vertical magnetic field and are thus more consistent with our previous assumption of a vertical magnetic field.

The use of a two dimensional profile analysis seems valid as most of the major structural features in North America are elongated to the north or northeast. Among these features are the Appalachian Mountains, the Appalachian Basin, the Midcontinent gravity anomaly, and the Colorado Front Range.

The profiles are based on Woollard's observed Bouguer gravity values and his observed magnetics. (Figure 13). Therefore, no regional scale anomalies have been removed. Both the gravity and magnetics were sampled at a 10 km sampling interval. It should be pointed out that at regions between 400 and 800 km the profile departs significantly from its usual east-west trend which may have some effect on the results of the analysis.

Several internal correspondence runs were made with various window sizes and the results of an analysis using a window size of 150 km is presented in Figure 14. The data are somewhat more irregular in character than our previous model studies but several regional scale interpretations are attempted.

Between 0-300 km, which is primarily over Southern California, there appears to be a region characterized by positive correlations and slope coefficient values ranging from 1.0 to 3.0. The primary sources for these positive relationships appear to be the San Joaquin Valley and a zone of positive gravity and magnetic anomalies along the western edge of the Sierra Nevada Mountains. The region between 300

and 800 km on the profile is characterized by a rapidly changing correlation coefficient values but appears to be predominantly negative. The slope coefficient values seem to range between +1.0 and -6.0. This region is underlain primarily by the Basin and Range Province.

Between 800 and 1000 km along the profile there is a zone of positive correlation with corresponding slope coefficients of approximately +1.0. This region contains the northwestern portion of the Colorado Plateau. The northeastern margin of the Colorado Plateau is characterized by a weak negative correlation. Between 1200 and 1500 km there is a region of generally positive correlation with slope values ranging from 0.0 to about +6.0. This region includes the Colorado Front Range and the Denver Basin. The negative correlation at 1600 km is not reflected by any surface geological feature.

From 1700 to 2200 km there is a general tendency for positive correlation and slope coefficients with this region having a fairly consistent value of about +2.0. The western great plains and the Midcontinent gravity and magnetic anomaly fall within this area. Between 2200 and 2400 km a weak negative correlation between gravity and magnetics exist and the slope coefficients reach values below -20.00. This area may correspond to what Zietz and others (1966) have called the "Eastern Iowa Magnetic Area" which lies to the north of this profile. Within this area, several large scale magnetic anomalies occur with little corresponding variation in the gravity field.

The area between 2600 and 3400 km is a zone of generally positive correlation with slope coefficients that vary from -5.0 to +6.0.

Between 3400 and 3600 km there is a negative correlation with slope values of as low as -15.0. This negative area corresponds to what Zietz and others (1966) has called the "Central Ohio Magnetic Area".

The western boundary of this magnetic area, Zietz and others have suggested may be the southern extension of the Grenville front in Canada. This region would occur at about 3200 km on Woollard's profile. From 3600 to 400 km the correlation is generally positive with slope coefficient values of between +2.0 and +5.0. This region includes the Appalachian Basin and the Appalachian Mountains.

The scatter diagrams of the transcontinental profile are more irregular in pattern and harder to interpret than the theoretical data (Figure 15). The effect of isostasy tends to scatter the diagrams along the gravity axis with the western states generally occurring to the left.

In general, internal correspondence analysis of Woollard's transcontinental profile indicates that this method is potentially valuable for the identification and interpretation of geologic provinces.

Cluster Analysis

Introduction

During the combined analysis of gravity and magnetic data, the interpreter often outlines regional areas where the magnetic and gravity relationships are similar and relates these to regional geologic provinces. A good example of this work is Lidiak's (1971) work on the basement of South Dakota. This visual process resembles cluster analysis. A brief review will be made of this technique and its

possible applications to the semi-quantitative analysis of gravity and magnetic data.

Cluster analysis involves the measurement of several variables in each of a group of samples. The similarity between the samples based upon a comparison of their variables is determined with some statistical parameter, usually a correlation coefficient. Thus, the initial group of samples is broken down into smaller groups of similar samples. Cluster analysis has been successfully used by paleontologists for years and its potential uses in other fields of geology are currently being realized.

The application of cluster analysis to gravity and magnetic data are straight forward. An area of gravity and magnetic data can be divided into several subareas representing separate samples. A variety of traits describing the gravity and magnetic data can be determined for each subarea and subsequently undergo cluster analysis. An analysis as described could be a significant improvement for regional geologic interpretation using gravity and magnetic data. The use of cluster analysis is anticipated to be superior to visual clustering in that it can consider several variables at once over a whole range of subareas; a rather difficult task by any visual process.

Technique

The clustering computer program is based on an algorithm given in Davis (1973). This program uses a correlation coefficient to determine similarity between equally weighted variables. Digitized gravity and magnetic profiles and a specified window size are input into the program.

The distance along the profile is divided up into non-overlapping segments of the specified window size. The pre-determined variables are measured within each segment and the clustering is conducted. The program outputs a dendrogram consisting of joined groups of subareas (Figure 16). The axis describing the significance of each junction on the dendrogram is simply a correlation coefficient, the most similar samples are joined at a high positive correlation value.

Seven variables were selected for this preliminary study of clustering. These variables are the mean gravity value, the mean magnetic value, the variance of the gravity, the variance of the magnetics, the correlation coefficient between the gravity and magnetics, the regressed slope coefficient between the gravity and magnetics and finally, the regressed intercept coefficient between the gravity and magnetics. The first four variables are independent to either gravity or magnetics only and the final three concern a combined linear relationship between them.

Analysis of Profile Two

This profile was previously described in the section on internal correspondence. Several runs were made with different window sizes and the results of an analysis using a 22.5 km window is shown in Figure 17. A series of samples was considered a cluster if they joined at or above a correlation value of 0.5 on the dendrogram.

Although the results of clustering are far from perfect in this preliminary analysis, in an overall view they are encouraging. There

is a definite tendency for clusters to be associated with distinct lithologies. The segments for one cluster group, those designated by 3's, are associated with the granite bodies. Another cluster of segments, those designated by 6's, are associated with the basaltic dikes. A third cluster of segments, designated by 4's is associated with both the diorite and basalt bodies.

Analysis of Woollard's Transcontinental Profile

Cluster analysis was conducted on Woollard's transcontinental profile which is described in the section on internal correspondence. An analysis using a window of 250 km is presented in Figure 18. Once again, a series of segments was considered a cluster if they joined above a correlation value of 0.5.

The largest cluster on the profile, designated by 7's in the diagram, is associated with the midwestern craton. One segment of this cluster falls over the Midcontinent Anomaly area. A second cluster of two subareas, designated by 8's, are from geographically separated segments that may correspond to Zietz and others (1966) Central Ohio and Eastern Iowa Magnetic Areas. The Grenville front has been interpreted by Zietz and others at approximately 3200 km on Woollard's profile. There is a noticeable change in the clustering pattern near this position. Another small cluster of two subareas, designated by 3's on the figure correlate with the Basin and Range Province plus the western margin of the Colorado Plateau.

Analysis of Lake Huron Maps

As a further test of cluster analysis, an investigation was performed on a set of gravity and magnetic maps. The data (Figures 19 and 20) are taken from a survey of Lake Huron (O'Hara and Hinze, 1972) whose location is shown in Figure 20. The area is 370 by 170 km and is sampled on a 10 km grid. The gravity data were rounded to the nearest milligal and the magnetics to the nearest 10 gammas. The conversion of the profile clustering routine to maps is straight forward; instead of a segment and a profile, square subareas and a map are used. The same variables used in the profile analysis were used in this test.

The results of a 50 by 50 km subarea cluster analysis is shown in Figure 20. The geology of this map is based primarily on interpretations by O'Hara and Hinze (1972) on the gravity and magnetic data. For this analysis a group of subareas was considered a cluster if they joined at a correlation level of 0.24 or higher.

One cluster specified by 3's on the figure is associated with the areas immediately west of the interpreted Grenville front which is a portion of the ^kPenokean Province. A second cluster occurs in areas immediately east of the proposed position of the Grenville front and are designated by 5's in Figure 20. A small cluster of two subareas, designated by 4's on the figure, fall directly on the interpreted position of the Grenville front. A cluster group delineated by 2's tends to favor areas within the Grenville Province.

Conclusions

Two possible approaches to semi-quantitative analysis of gravity and magnetic data have been discussed and have been applied with encouraging results. The correlation of magnetics and gravity using internal correspondence and clustering are shown to aid in the interpretation and mapping of regional geophysical parameters and delineation of geologic sources and provinces. However, both procedures are in their infancy and much work remains before they can be effectively applied to geophysical interpretation.

IV. UTILITY OF POISSON'S THEOREM IN MAGNETIC AND GRAVITY ANALYSIS

Theory

The gravitational potential U , at an exterior point, due to the mass of a body of uniform density σ can be related to the magnetic potential V due to the same body polarized uniformly in the direction i with an intensity of magnetization J by the theorem attributed to Poisson (1826)

$$V = \frac{J}{G\sigma} \frac{\partial U}{\partial i} \quad (1)$$

where G is the gravitation constant (6.67×10^{-8} cgs units). Thus the magnetic potential and the derivative of the gravitational potential are related linearly by a constant factor $J/G\sigma$.

It is important to emphasize the assumptions under which Poisson's theorem is valid. It is assumed that the potentials U and V are due to a common causative body which has a uniform density σ and magnetization J (in both intensity and direction of magnetization) and, the ratio $J/G\sigma$ and the inducing magnetic field H are constant over the entire area of potential fields. Usually the anomalous potentials are used from which the earth's main potential fields have been removed and local variations are analyzed in terms of density and magnetization contrasts. It is also important to note that the validity of Poisson's theorem is not dependent on the shape or the depth of the causative bodies with the minor exception of the effects of the demagnetization factor for bodies whose surfaces are very irregular.

If the vertical component of the magnetic field is measured, (1) may

be differentiated to obtain

$$Z = \frac{\partial V}{\partial z} = \frac{J}{G\sigma} \frac{\partial^2 U}{\partial z \partial i} \quad (2)$$

or, interchanging the order of differentiation

$$Z = \frac{J}{G\sigma} \frac{\partial^2 U}{\partial i \partial z} = \frac{J}{G\sigma} \frac{\partial \Delta g}{\partial i} \quad (3)$$

where $\Delta g = \frac{\partial U}{\partial z}$ is the vertical component of gravity that is measured by the gravimeter.

If instead of the vertical magnetic field, the total field ΔT is measured, then two additional assumptions are necessary in order to convert (1) to a workable form. Differentiating (1) in the direction of the total field, we obtain

$$\Delta T = \frac{\partial V}{\partial i} = \frac{J}{G\sigma} \frac{\partial^2 U}{\partial i^2} \quad (4)$$

For this equation to be valid the anomalous field ΔT due to both induced and remanent effects must be small relative to the earth's main field. Fortunately, except for very large local disturbances (on the order of 10,000 gammas) this condition is satisfied. Furthermore (4) requires that derivatives of U be known. U may be calculated approximately (Cordell and Taylor, 1971) or the spatial derivatives of U approximated directly (Kosbahr, 1949) by integration of Δg . This operation is valid only if Δg is small relative to the main earth's gravitational field; again a condition which is usually satisfied in practice.

Equations (3) and (4) are useable expression of Poisson's theorem and are linear equations relating measured values of the magnetic field to derivatives of the gravitational field. Assuming that sufficient

observations of the gravitational and magnetic fields are available over a finite homogeneous source body, then (3) and (4) are over-determined equations having three independent unknowns, J/σ and the direction of J which can be expressed most easily by inclination I_t and declination D_t of the total magnetization vector. Here it is assumed that the inducing field strength H and its direction (I_o, D_o) are known. J is actually the length of the total magnetization vector which is the sum of induced and remanent magnetization vectors of length kH and J_r and direction (I_o, D_o) and (I_r, D_r) respectively, where k is the magnetic susceptibility.

It is clear that one cannot uniquely determine all of these quantities even given perfectly accurate magnetic and differentiated gravity observations. Theoretically J/σ and the direction of J can be determined uniquely, given the assumptions implicit in applying Poisson's theorem. If we assume remanent magnetization is negligible then $J = J_o = kH$ and given certain bounds on σ the range of k may be calculated from

$$J/\sigma = (G \frac{\partial^2 U}{\partial i^2} / \Delta T)$$

or

(5)

$$J/\sigma = (G \frac{\partial \Delta g}{\partial i} / Z)$$

Since J is the length of the vector sum of the induced and remanent magnetization, both of these vectors cannot be uniquely determined. However their lengths are related by the Koenigsberger ratio $Q = J_r/kH$, the ratio of remanent to induced magnetization in a rock. Various values

of J_r and kH and their associated directions may satisfy Poisson's theorem. However the minimum value of $Q = J_r/kH$ is found when the remanent and total magnetization vectors are perpendicular (Grossling, 1967; Cordell and Taylor, 1971). Therefore, solving equations (3) or (4) can yield, theoretically, unique determinations of J/σ , the inclination and declination of the total magnetization vector (J), and the minimum Koenigsberger ratio.

Applications

Poisson's theorem was expressed explicitly by Eotvos (1907) in terms of the components of the magnetic field relative to the second derivatives of the gravitational potential which are measured by the torsion balance. Haalck (1929) applied the equations given by Eotvos to torsion balance and magnetic observations in the Kursk area of Russia and determined the density-susceptibility ratio of the anomalous body. Garland (1951a, b) extended Poisson's theorem to vertical and total magnetic field and gravimeter measurements. Garland (1951b) applied Poisson's theorem to the Crow Lake anomaly in the Canadian shield by calculating the theoretical total magnetic field from the gravity data assuming uniform density and magnetization and comparing with the observed magnetic data. The results indicated dramatically that the source body was of non-uniform magnetization. However the difference between the calculated and observed magnetic field data served to delineate the separation between two rock types which provided the source of the anomalies. The rock types were of uniform density but differed markedly in their magnetization. Using the simpler and more isolated Marwora

anomaly, Garland estimated the magnetization/density ratio using the total field magnetic anomaly and derivatives of the gravity data calculated at six locations. Remanent magnetization was assumed to be negligible and the density contrast was estimated to be 0.3 g/cc. The results indicated that the assumption of uniform magnetization, and negligible remanence were at least compatible with the observations. A common source for the gravity and magnetic anomalies was therefore indicated, having a J/σ ratio of $90,000 \times 10^{-6}$.

Garland (1951a) also determined J/σ for an anomaly in Arkansas and attempted an interpretation of the range of J/σ values in terms of rock type. Garland also mentions the importance of removing regional gravity and magnetic fields, removing the effects of neighboring bodies and testing for uniform magnetization.

Nettleton (1942) derived several formulas for gravity and magnetic calculations over single bodies such as spheres and cylinders. Although Nettleton does not refer to Poisson's theorem, he expresses the relationship between the gravity and magnetic fields over the center point of single bodies (for vertical field and vertical magnetization) such that J/σ can be determined from the peak amplitudes of the gravity and magnetic fields. The formulas given by Nettleton may be used for quick estimates of Poisson's relation and determination of J/σ since no derivatives are needed. This approach requires an assumption of source geometry and depth.

Several authors have investigated the determination of the direction of magnetization of a body using some variation of Poisson's theorem. Lundbak (1956), Ross and Lavin (1966), Bott and others (1966) and Robinson (1971) have determined the magnetization direction for two- and three-dimensional theoretical and real bodies by successively transforming the

gravity or magnetic fields according to Poisson's theorem assuming values of the inclination and declination of the magnetization vector. The direction of magnetization is selected on the basis of the best fit between the transformed and the observed field. If the susceptibility and density of the causative body are assumed, then limits can be placed on the range of compatible remanent magnetization vectors. The remanent directions thus determined have been applied to paleomagnetic studies (Lundbak, 1956; Ross and Lavin, 1966). The importance of the estimation of the base level of the anomalies and removal of regional gradients has been emphasized by Ross and Lavin (1966) and Bott and others (1966).

Baranov (1957) has employed Poisson's theorem to derive a pseudo-gravity field in which the magnetic field is effectively "reduced to the north magnetic pole" assuming a direction of magnetization and the ratio J/σ . The effects of asymmetry of magnetic anomalies due to inclination of magnetization is thus removed and the pseudo-gravity data are much easier to correlate with observed gravity for subsequent interpretation.

Recently, Kanasewich and Agarwal (1970) have applied modern digital processing techniques to Poisson's theorem to provide a statistically significant determination of J/σ . The analysis is carried out in the wave number domain. Both the gravity and magnetic data are transformed to the wave number domain, filtered to remove short wavelength noise, the gravity data upward continued to the flight elevation of the aeromagnetic data, and the necessary horizontal and vertical derivatives of the gravity field determined. The observed magnetic data are reduced to the

pole before comparison with the transformed gravity data. It is assumed that remanent magnetization is negligible and that the magnetization vector is constant in both direction and length over the entire area. A broad region encompassing many anomalies is used in the hope of statistically enhancing the J/σ estimate. J/σ is determined for each wavelength by dividing the Fourier amplitudes of the magnetic field by the differentiated gravitational field for each wavelength. J/σ may be plotted against wavelength and an average value determined. The distribution of J/σ as a function of wavelength also may contain valuable although not readily interpretable information. In order to aid in the selection of a representative J/σ value for the area, the coherency of the magnetic data and the differentiated gravity data is calculated. Thus for each wavelength a coherency between the gravity and magnetics and an estimate of J/σ is determined.

There are several difficulties with the approach of Kanasevich and Agarwal to Poisson's theorem. First, all spatial domain information is lost since the J/σ and coherency estimates are accomplished in the wave number domain. This is a serious problem whenever more than a single, isolated anomaly is treated since the gravity information of a particular wavelength from one section of the map is included in the analysis with the magnetic information of the same wavelength from an entirely unrelated part of the map. Furthermore, since a broad area is used, the assumption of uniform density contrast and magnetization over the map is especially suspect. However, the Kanasevich and Agarwal approach cannot be applied on a very small area since the entire anomaly must be included and the number of data points must be large enough (relative to the dominant

wavelength of the anomaly) for adequate wave number estimates to be made by the Fourier analysis. No provision is made for remanent magnetization except as can be included in trial and error calculation of the transformed gravity data, and even in this case the magnetization vector must be assumed constant over the entire map.

Perhaps the most complete numerical approach to the application of Poisson's theorem has been developed by Cordell and Taylor (1971). These authors determine J/σ and the direction of the magnetization vector by a least squares inversion of the gravity and magnetic data. Basically equation (4) is used and the observed data are transformed to the frequency domain. An estimate of U is found by expressing U as the integral of Δg in the frequency domain. A linear system is expressed in which the unknowns are the components of the total magnetization vector divided by σ and the known quantities are the inducing magnetic field strength and direction and the Fourier transformed observations of the gravity and magnetic fields. The equation is greatly overdetermined and is solved by the method of least squares at times weighting the solution by using only certain wavelengths of the transformed data.

Cordell and Taylor applied the method to a theoretical anomaly with excellent results. Application to an isolated gravity and magnetic anomaly over a seamount was also successful. Determination of J/σ by this method was shown to be highly accurate so long as the assumptions implicit in Poisson's theorem are met. Using reasonable estimates of density the range of susceptibility contrast may be estimated as well as the

range of possible remanent magnetization directions and intensities and the minimum Q . Paleomagnetic pole positions were then calculated based on the possible Q values for the seamount.

Although the method presented by Cordell and Taylor appears to be very powerful, it is still subject to several limitations partly imposed by the assumptions of Poisson's theorem and partly by the numerical techniques used. The anomalies considered must be isolated so that the spectral amplitudes are representative of the fields due to a single body and so that the base level of the anomalies may be determined satisfactorily. Numerical estimates of the spectrum of an anomaly are also inadequate unless the available data covers an area which is large relative to the dominant wavelength of the anomalies.

Application of Poisson's theorem has proved to be of significant value in geological and geophysical interpretation, and the method appears to have potential for greater emphasis if the many limitations imposed by the theory and the numerical application can be reduced. These difficulties are summarized and evaluated below.

1) The validity of Poisson's theorem is dependent on gravitational and magnetic anomalies arising from a common, finite homogeneous source having uniform density and magnetization. While these assumptions are seldom if ever satisfied in practice, the results of application of Poisson's theorem may yield significant results even if reliable values of J/σ and the direction of magnetization cannot be given. Comparison of magnetic and transformed gravity data may be used to delineate zones of anomalous J/σ or magnetization as shown by Garland (1951b).

2) Interference of gravity and magnetic anomalies due to small separation between source bodies will reduce the effectiveness of the Poisson analysis if the bodies differ in density or magnetization. Real data nearly always will be contaminated by the effects of several sources or at least regional anomaly fields. Wilson (1970) presents a method based on Poisson's theorem which allows the separation of the anomalous gravity and magnetic fields of neighboring bodies. However, the method requires the assumption of the number of source bodies present, the value of J/σ and the direction of magnetization for each body. Wilson's method may have advantages in the accurate modeling of a single anomaly since the individual fields can be separated, but as an aid to the Poisson's theorem analysis it involves too many assumptions about the bodies. One of the approaches to a better application of Poisson's theorem in the presence of interfering anomalies is to attempt to apply the method to the central portion of the anomaly. This will probably require a spatial domain approach and particular attention paid to the base level of the anomaly as discussed below. If a method for Poisson's analysis using a relatively small portion of an anomaly could be developed, the technique could be applied successively over a large map area yielding nearly continuous estimates of J/σ and direction of magnetization.

3) It is clear from the form of equation (5) that the base level or regional gradients of the gravity and magnetic fields will significantly influence the J/σ determination. Gradients will especially affect the gravity data since horizontal derivatives are necessary unless the direction

of magnetization is vertical. These effects can be minimized by removing known base levels or gradients. When an anomaly is not isolated, the base level is difficult to determine and the central portion of the anomaly may be all of the data that is useful in the analysis. In this case if one plots the magnetic data versus the differentiated gravity data, the slope of the resulting line will be $J/G\sigma$ which will not be affected by the base level.

4) Several numerical techniques are necessary for the application of Poisson's theorem. Vertical and horizontal derivatives of the gravity data must be adequately determined. Vertical gradients have commonly been calculated by Baranov's (1953) formula and horizontal derivatives by simple difference methods. Alternatively, all derivatives could be evaluated by wave number domain methods as described by Bhattacharyya (1972). The possible extent of inaccuracy of Poisson's analysis due to numerical derivative operations is not presently known. Gravity data must be upward continued to the elevation of the magnetics for application of Poisson's theorem. Upward continuation acts as a wavelength filter and tends to remove a certain amount of noise.

Upward continuation of both the gravity and the magnetics, for example to satellite elevation, may be a desirable approach to Poisson's theorem in that interfering anomalies and fields due to bodies having non-uniform density and magnetization will be smoothed and averaged. A representative value of J/σ and direction of magnetization might then be determined for a broader region although the exact averaging process that would determine J/σ and direction of magnetization is not presently known.

Until a thorough study of the application of Poisson's theorem to theoretical and real gravity and magnetic data comprising a wide range of geological and geophysical conditions and an analysis of the effects of various numerical techniques is made, the resolution and applicability of Poisson's theorem will remain unknown and the method will be restricted to a limited range of rather simple geologic applications.

V. GEOLOGIC IMPLICATIONS OF COMBINED MAGNETIC AND GRAVITY ANALYSIS TECHNIQUES

The objective of combined magnetic and gravity analysis is to map geologic and geophysical characteristics of the lithosphere and to decipher geologic history from these data. This information is important not only to the basic understanding of the earth and its processes, but also to the solution of environmental and mineral and petroleum exploration problems. Thus the results of the suggested, but largely untested numerical techniques for magnetic and gravity analysis must be related to parameters of the earth. This at least in part can be accomplished by extending correlations achieved by qualitative techniques and conjecture founded on fundamental geophysical and geologic concepts.

An important and direct use of combined magnetic and gravity analysis is the identification of source lithology. Nettleton and Elkins (1944) have determined the ratio of magnetization (induced) to density for igneous rocks classified by the C.I.P.W. and Iddings methods. However, this ratio does not lead directly to identification of lithology because the magnetizations and densities of rocks generally overlap (Dobrin, 1960). This has been corroborated by physical property measurements of basement rocks in the central United States (Rudman and Blakely, 1965). The problem is further complicated by other problems leading to ambiguity, particularly the effect of remanent magnetization.

In spite of the difficulties of relating specific lithologies to gravity and magnetic anomalies, certain generalizations are possible.

These generalizations are based upon correlation of anomalies with direct geologic information from outcrops and drill holes. Mafic intrusive and extrusive rocks are generally associated with positive gravity and magnetic anomalies. However, locally both of these rock types may produce negative magnetic anomalies due to remanent magnetization. Granite intrusives generally cause negative gravity anomalies and either negative or positive magnetic anomalies depending on the nature of the country rocks. Over the Canadian Shield granitoid rocks and highly altered gneisses generally correlate with magnetic highs and belts of Precambrian sedimentary and volcanic rocks and low-grade gneisses correlate with magnetic lows, although the latter may contain numerous narrow magnetic highs. Within this area, gravity highs correlate with granitoid belts and highly metamorphosed volcanic-sedimentary formations, whereas regional gravity lows correspond to weakly metamorphosed volcanic-sedimentary formations. One of the richest ore deposits in the world is located within the Boulder batholith in Montana. The batholith is correlated with a broad gravity low and a magnetic anomaly maximum. Interestingly, the actual ore deposit shows up in a reverse sense, an intense magnetic minimum and a low-amplitude gravity maximum. In South Dakota Lidiak (1971) has found that mafic schists in the basement are characterized by gravity highs and less pronounced magnetic highs than are associated with the gneiss belts. The relationships between gravity and magnetic anomalies and rock type are obviously complex. However, even in the case of a positive gravity anomaly (the Mid-Michigan gravity anomaly) which along its strike

has correlative positive, negative and no magnetic anomalies, lithologic implications can be inferred from the correlations or lack thereof (Hinze and others, 1975).

Correlation of the characteristics of both gravity and magnetic anomalies may be extremely useful in mapping geologic provinces that reflect units of relatively homogeneous lithologic suite, tectonics, and geophysical parameters. Utilizing in part gravity and magnetic data Pakiser and Zietz (1965) have divided the crust of the United States into two major zones separated by the eastern edge of the Rocky Mountains. MacLaren and Charbonneau (1968) have discussed the magnetic and gravity patterns associated with the provinces of the Canadian Precambrian shield. Rudman and others (1965) and Hinze and others (1975) give examples of the identification of buried basement provinces in the Midwest using magnetic and gravity data. Identification and mapping of similar provinces over the entire earth may be possible utilizing satellite magnetic observations and world-wide gravity data. This would supplement and perhaps refine the correlations of gravity with world-wide tectonics (Kaula, 1972) and with plate boundaries (Wilcox and Blouse, 1974).

The geologic utility of combined gravity and satellite magnetic observations cannot be determined until the accuracy and resolution of the satellite observations is specified and analysis is performed incorporating upward continued aeromagnetic and gravity data and model studies. However, the potential is present for obtaining regional geologic information previously unavailable. This information may take the form of defining present plate boundaries or provinces which outline stabilized

plate boundaries on continents which relate to Pre-Mesozoic sea floor spreading. Broad zones of widespread hydrothermal alteration which accompany mineralization may be detected. Thermal plumes may be traced on the basis of decreased overall magnetization caused by temperatures exceeding the Curie temperature of magnetite. The high thermal inertia of the earth which causes temperatures within the earth to change very slowly may be used to detect previous positions and thus paths of thermal plumes in reference to the crust of the earth, these are only a few of the many exciting potential applications of combined gravity and satellite magnetic analysis.

It is clear that magnetic and gravity data at satellite observations will fail to resolve many of the types of anomalies that the geophysical analyst is accustomed to dealing with on ground or aeromagnetic maps. This will be a disadvantage. However satellite magnetic observations also have an advantage. An advantage because local perturbations in geology will not be observed at satellite elevations. These local anomalies are really noise in the interpretation of regional structures - noise which can seldom be extracted satisfactorily by filtering. Ore bodies are best studied from ground observations and batholithic sized features can best be investigated at aircraft elevations, but features such as the Colorado Plateau and its relation to the tectonics of southwestern United States is best studied at satellite elevations where the anomalies are free of noise due to local geologic features. Thus, satellite derived data is expected to aid in mapping regional structures and provide average properties of the lithosphere. These are objectives that are difficult to achieve with our present data base.

To achieve these goals and utilize the satellite magnetic data to the maximum degree, combined gravity and magnetic analysis of data must be performed. Only continued research will decide which of the methods outlined in this report, analytical spatial correlation, clustering or use of Poisson's theorem, will provide the optimum approach under varying geologic and geophysical conditions. Regardless of the method, quantified interpretational tools are urgently needed. The importance of these tools and research to refine them was foreseen by Davis (1973) when he stated "The subject of map comparisons is one which will become increasingly important in the future, because interpreting the voluminous data from Earth-sensing satellites will require development of automatic pattern recognizers and map analyzers. The algorithms which control these machines must be developed by geologists and other earth scientists, who alone have the knowledge of the Earth necessary to interpret the data. In turn, geologists must learn to quantify and systematize their mental recognition skills so that machines can be taught to assume some of the burden for them. If this is not done, we will be literally buried under reams of charts, maps, and photographs returned from resource survey satellites, orbiting geophysical platforms, and other exotic tools of the future."

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MAGNETIC
SUSCEPTIBILITY
CONTRAST
($\times 10^{-6}$ emu/cc)

DENSITY
CONTRAST (g/cc)

BODY

1	0.30	6000
2	0.30	-6000
3	-0.05	6000
4	0.05	-6000
5	0.00	6000
6	0.30	1000
7	-0.30	0
8	-0.05	1000
9	0.05	1000
10	0.00	1000
11	0.30	6000
12	0.05	6000
13	0.30	1000
14	0.05	1000
15	-0.05	1000
16	-0.30	1000
17	-0.30	0
18	-0.30	0
19	0.30	-6000
20	0.30	1000

TABLE 1. Density and magnetic susceptibility contrasts for bodies of Profile One.

<u>BODY</u>	<u>DENSITY CONTRAST (g/cc)</u>	<u>MAGNETIC SUSCEPTIBILITY CONTRAST ($\times 10^{-6}$ emu/cc)</u>
1	0.05	8000
2	0.05	8000
3	0.05	8000
4	0.05	8000
5	0.05	8000
6	0.25	14000
7	0.25	14000
8	0.25	14000
9	0.25	14000
10	0.25	14000
11	0.25	14000
12	0.25	14000
13	0.25	14000
14	0.25	14000
15	0.05	8000
16	0.25	14000
17	-0.10	3000
18	0.25	14000
19	0.05	8000
20	-0.10	3000
21	0.25	14000
22	-0.10	3000
23	-0.10	3000

TABLE 2. Density and magnetic susceptibility contrasts for bodies of Profile Two.

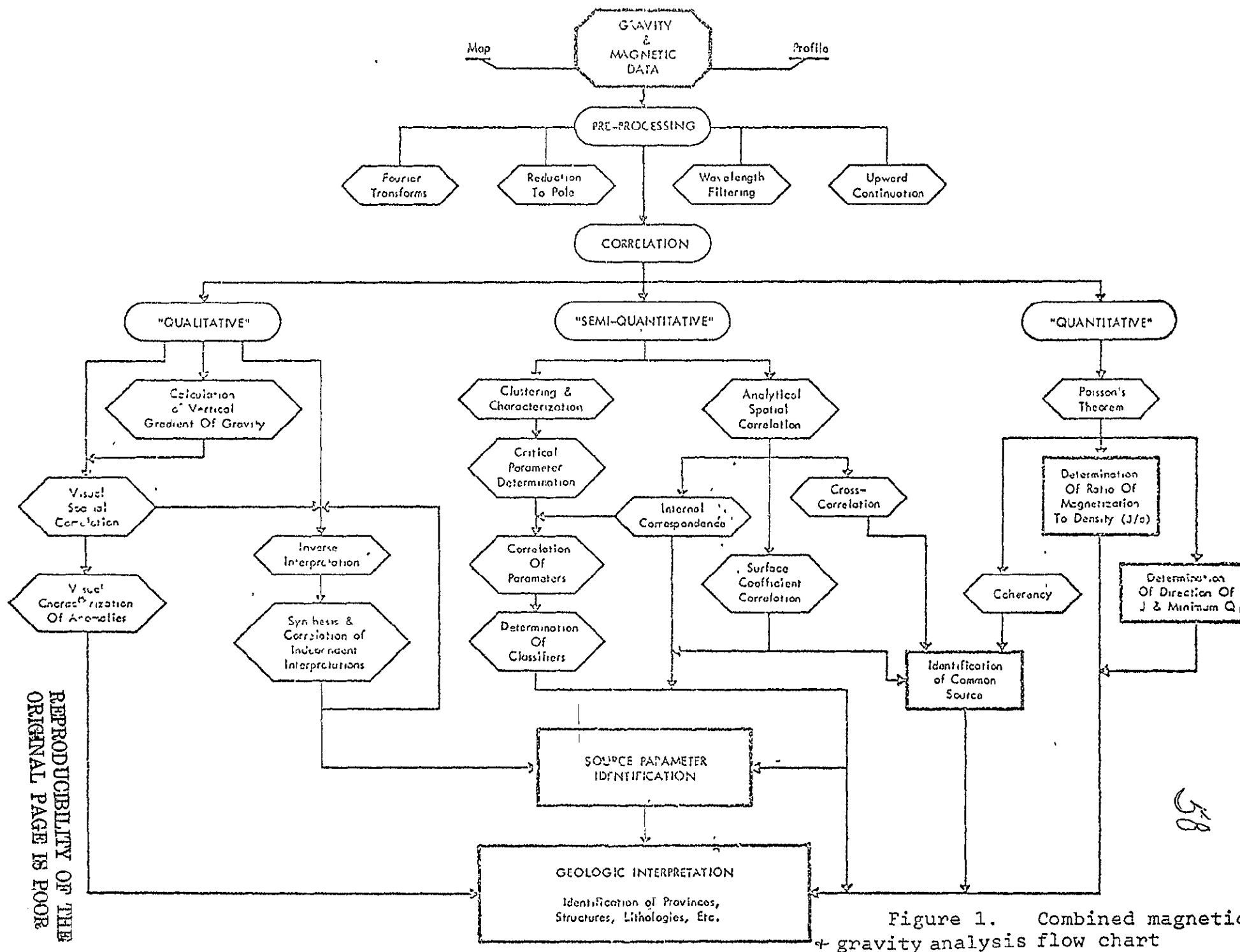


Figure 1. Combined magnetic + gravity analysis flow chart

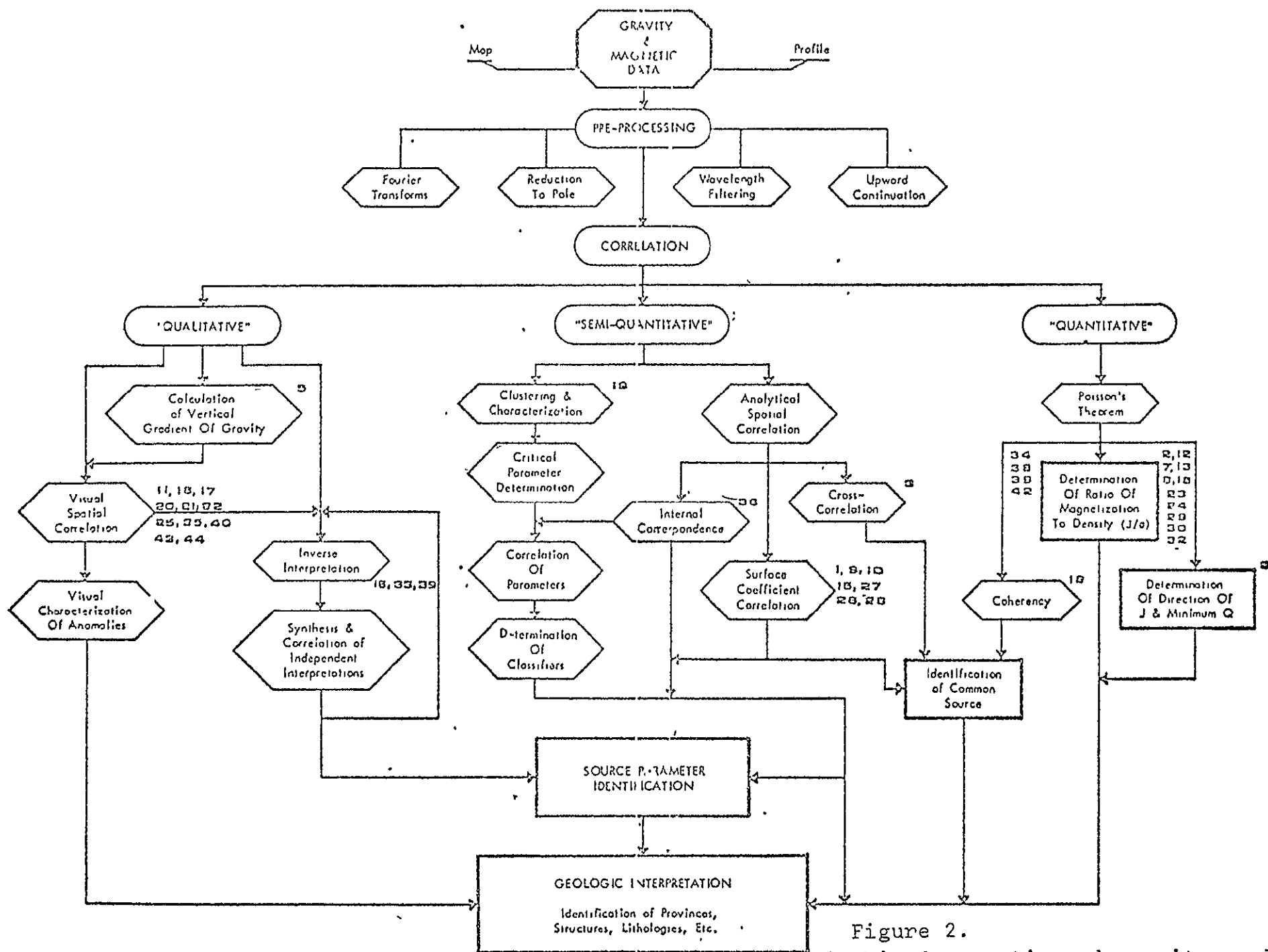


Figure 2.
Combined magnetic and gravity analysis
flow chart with reference numbers
keyed to annotated bibliography.

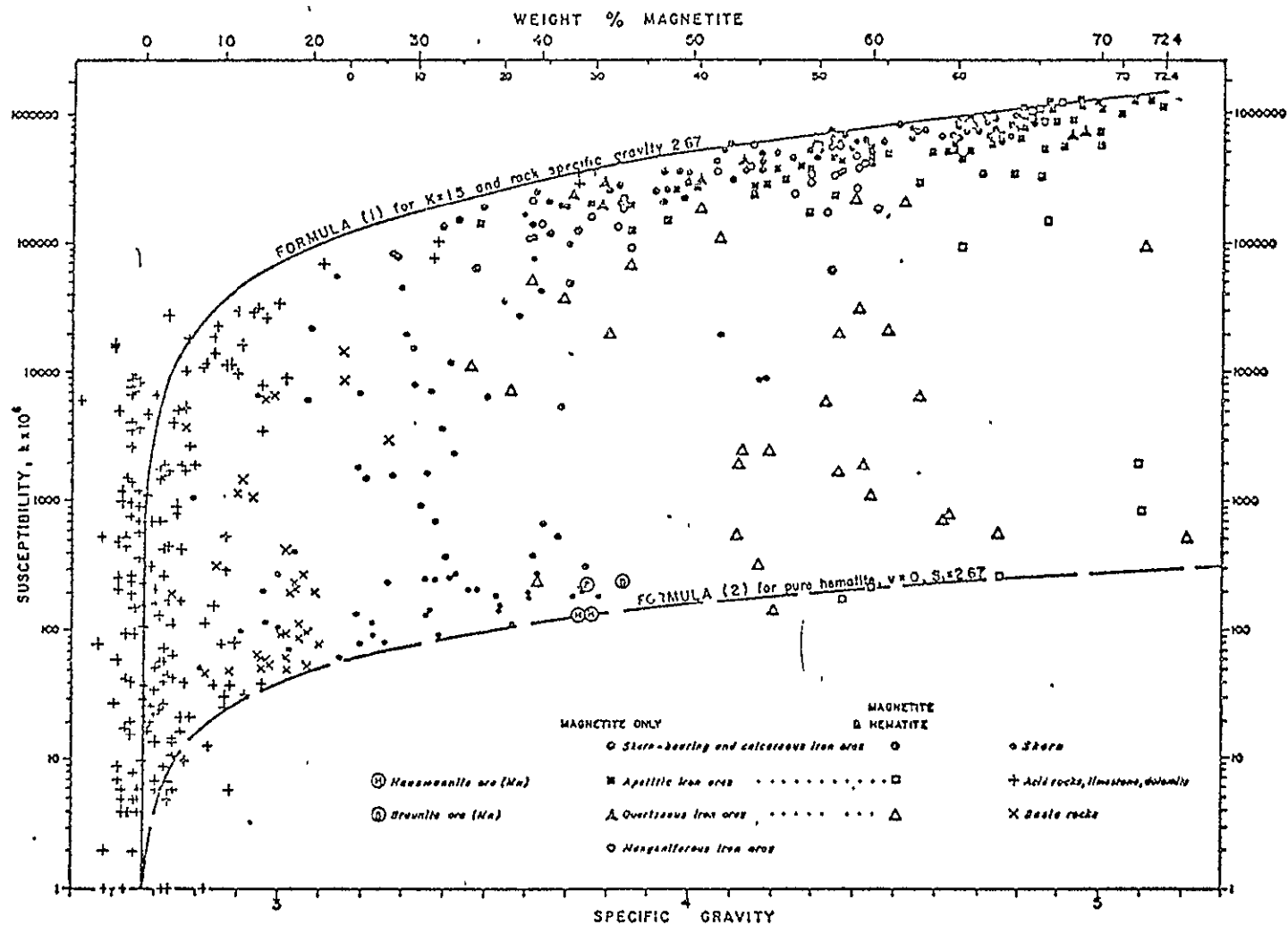


Figure 3. Magnetic susceptibility versus density (specific gravity) of selected Swedish rocks and ores (After Werner, 1945).

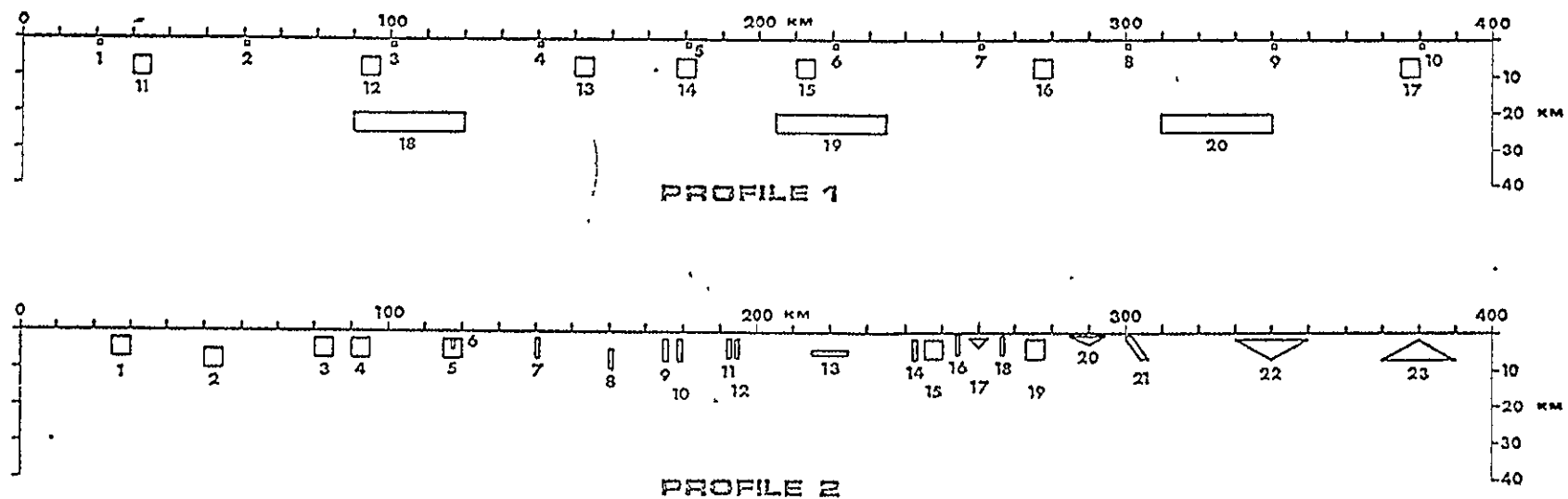


Figure 4. Cross-section of two-dimensional bodies of Profile 1 and 2. Density and magnetic susceptibility contrasts are listed in Tables 1 and 2.

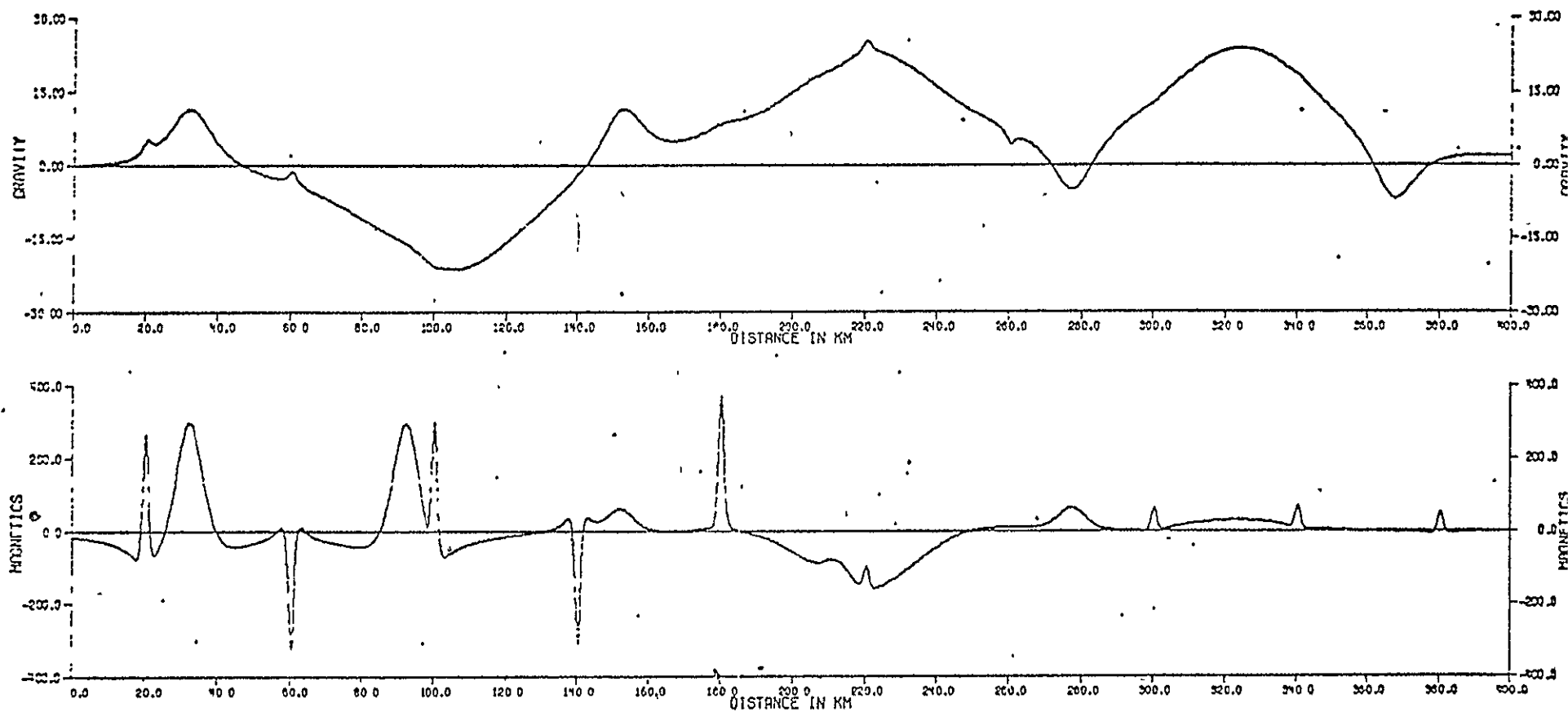


Figure 5. Calculated gravity and magnetic anomaly values for Profile 1.

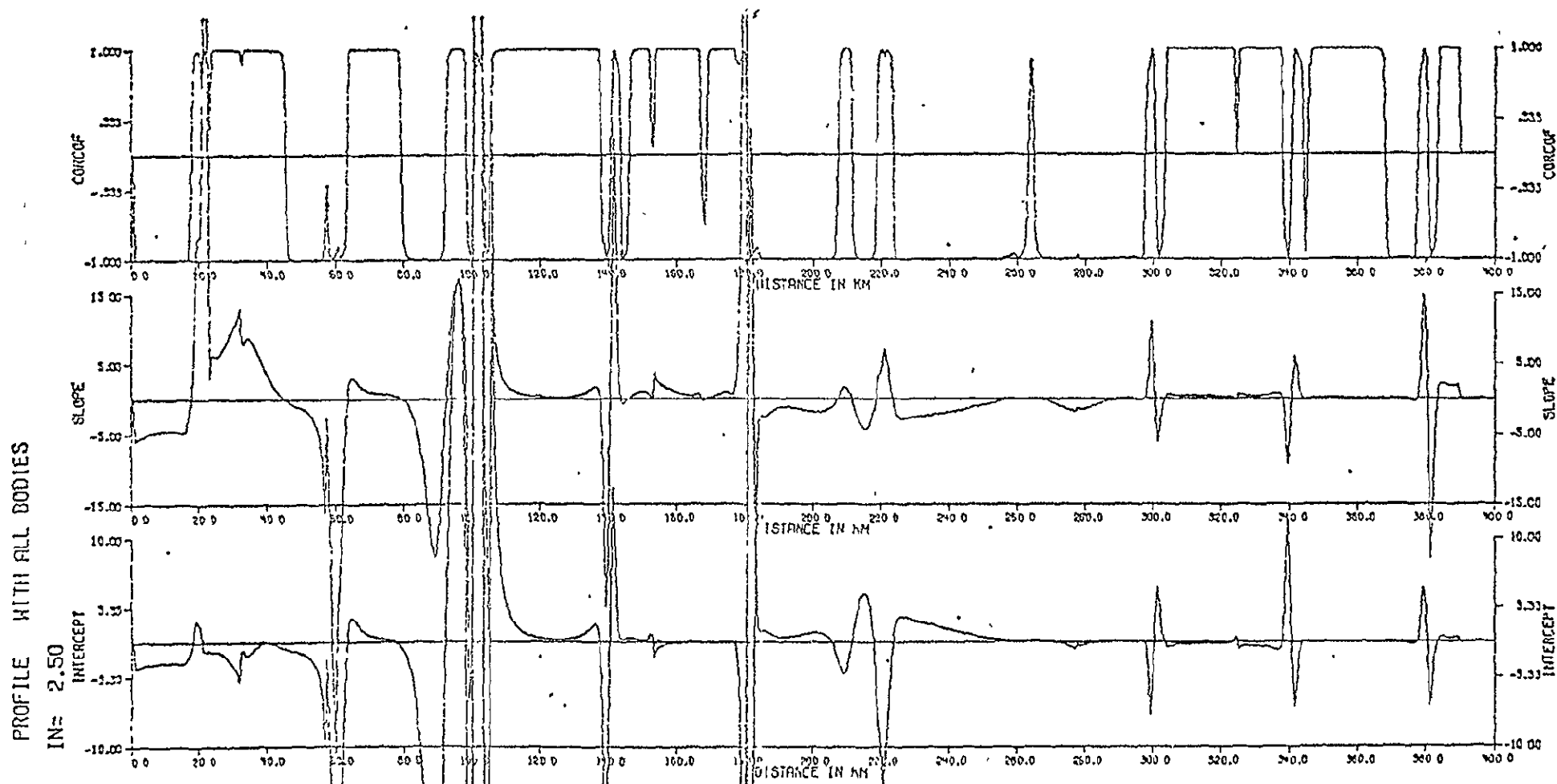


Figure 6. Internal correspondence results for Profile 1 using a window of 2.5 km.

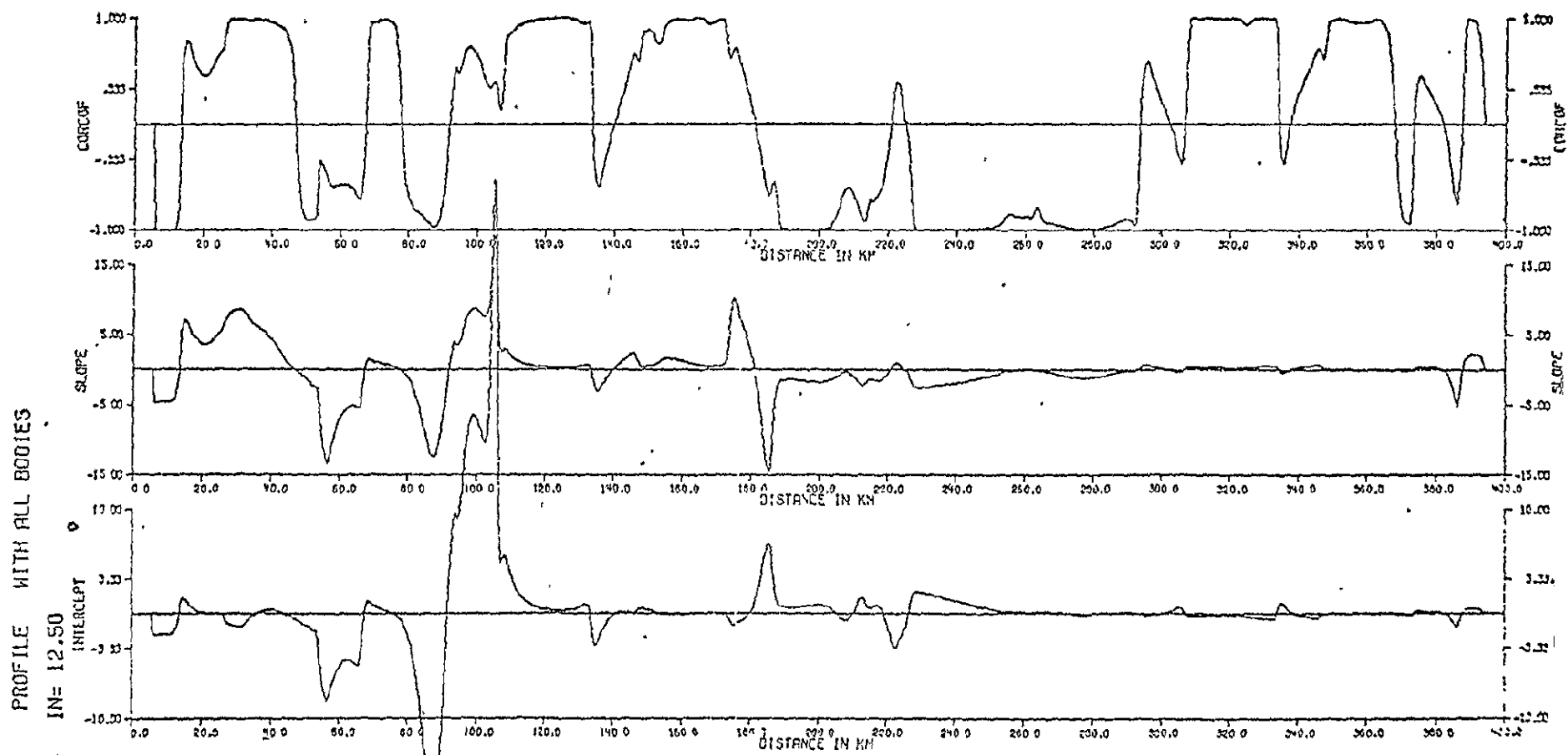


Figure 7. Internal correspondence results for Profile 1 using a window of 12.5 km.

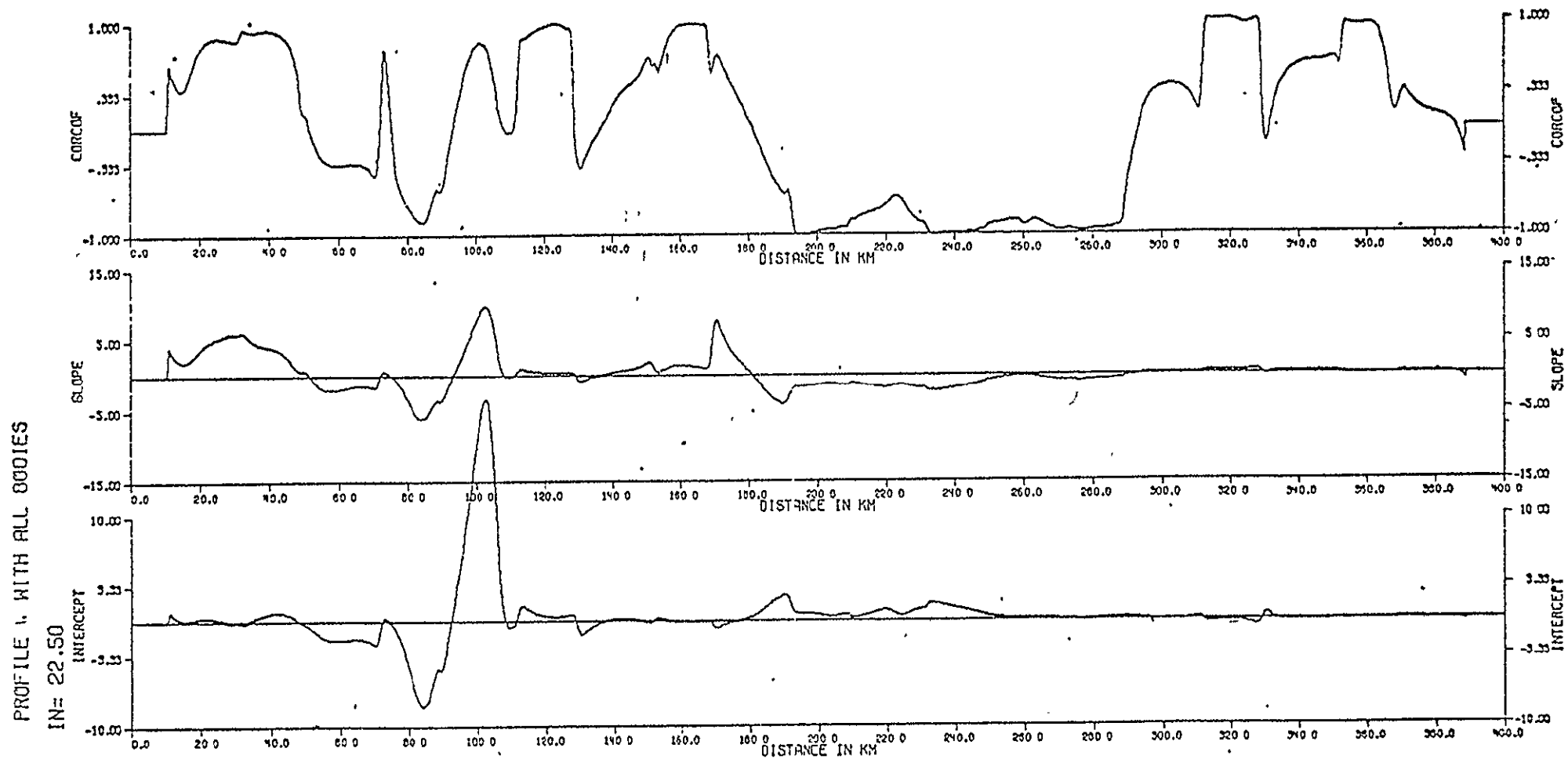


Figure 8. Internal correspondence results for Profile 1 using a window of 22.5 km.

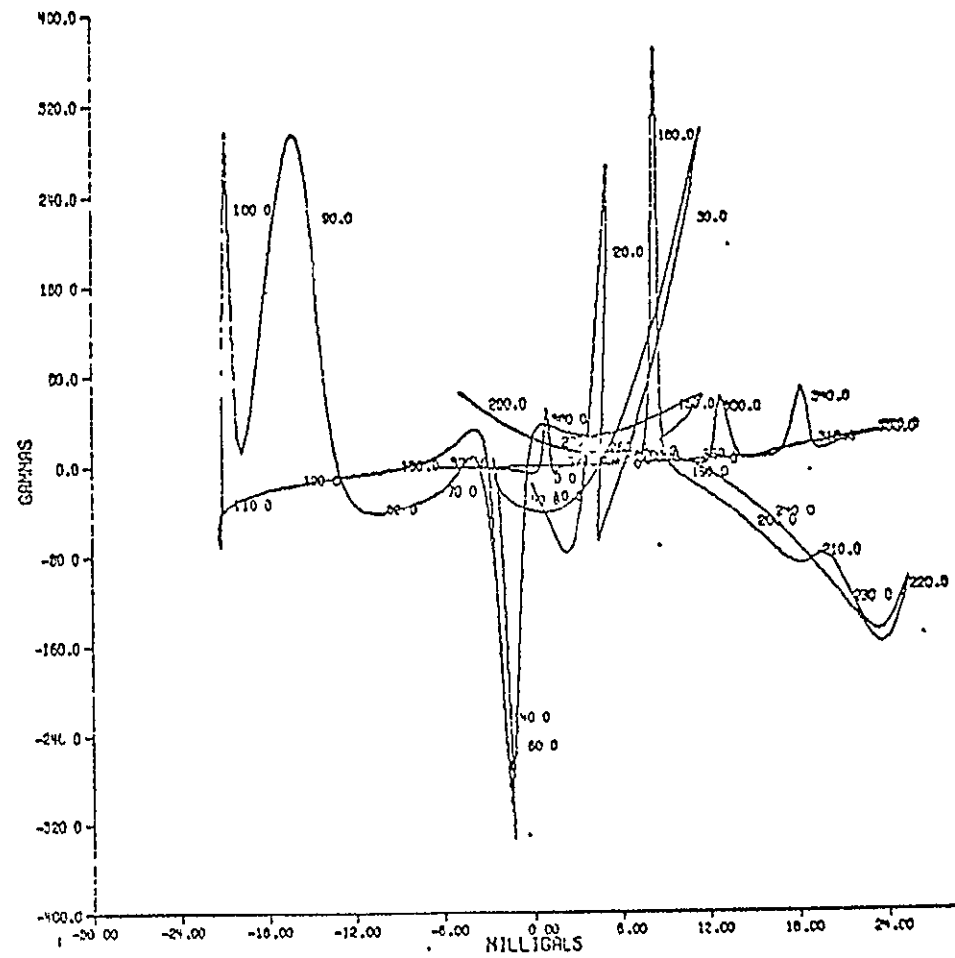
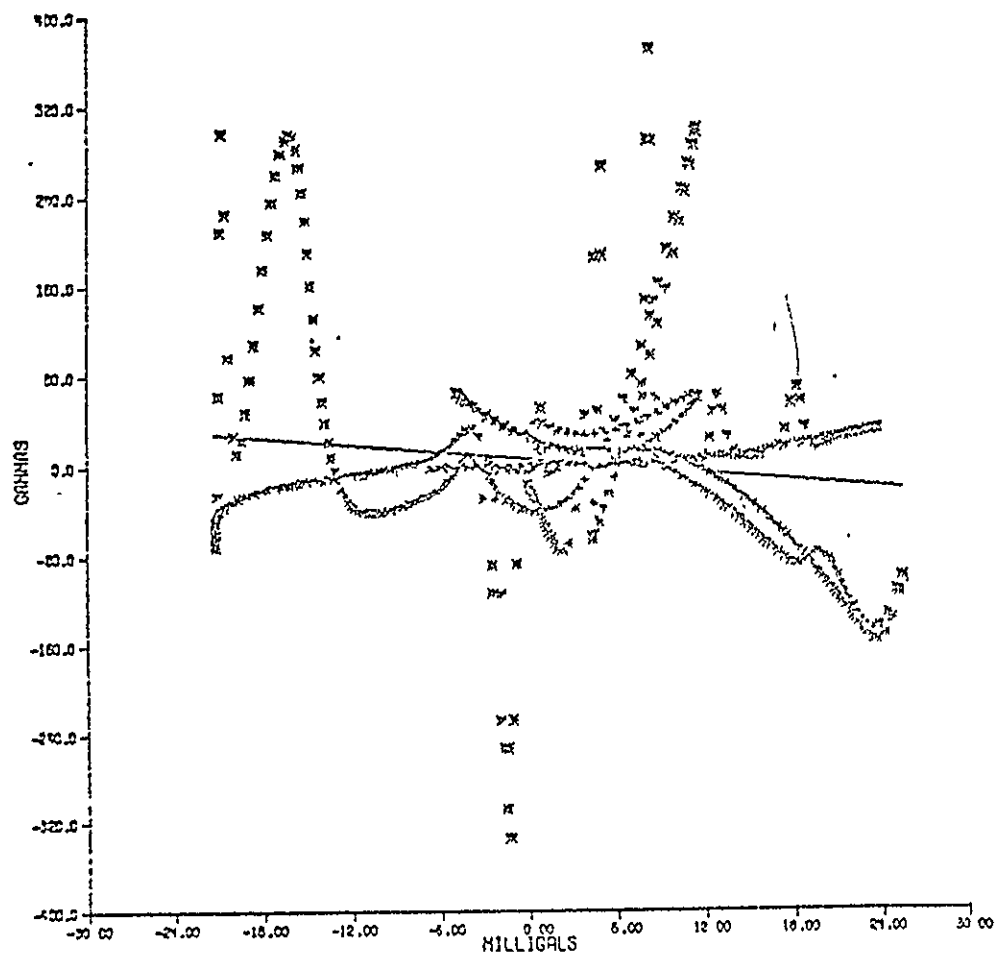


Figure 9. Scatter diagrams showing gravity vs. magnetic anomaly values for points along Profile 1.

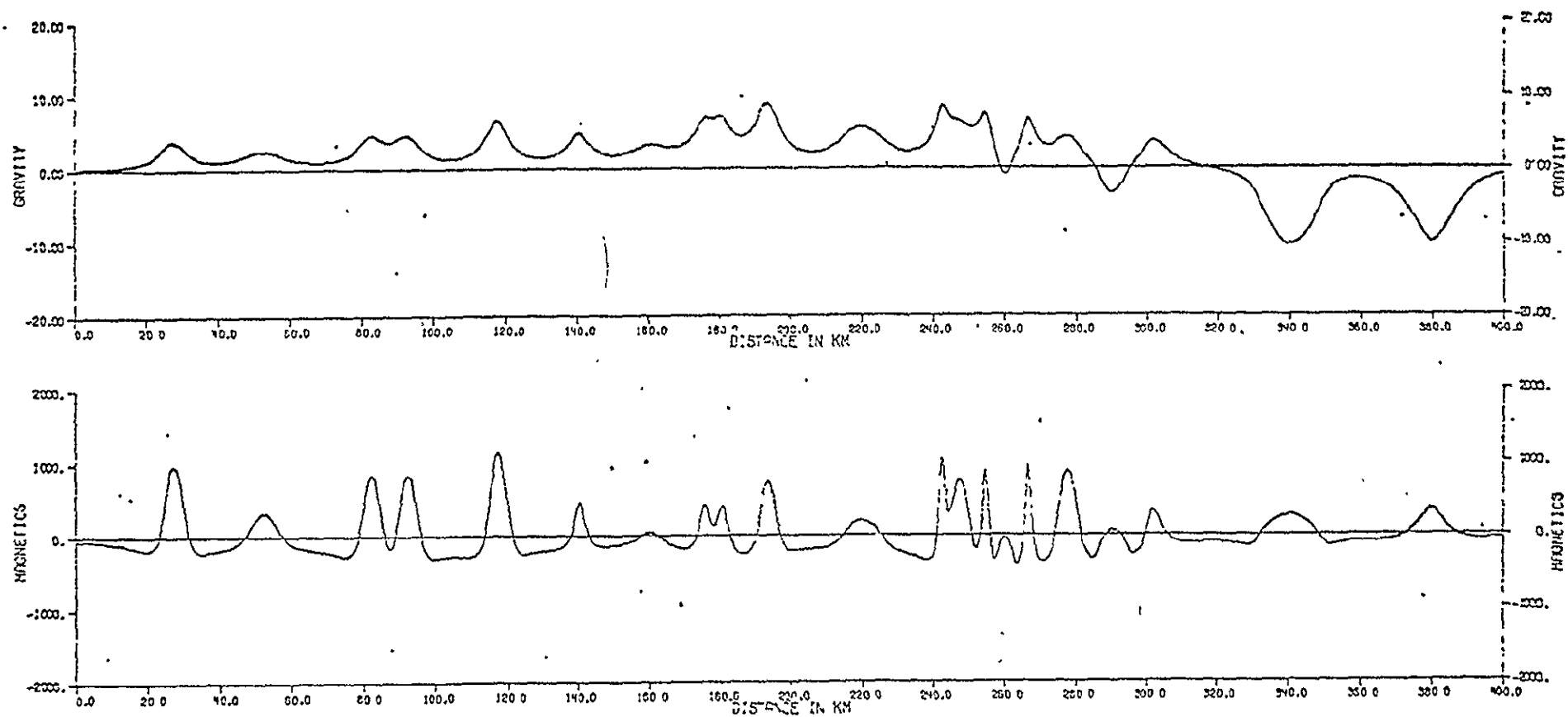


Figure 10. Calculated gravity and magnetic anomaly values for Profile 2.

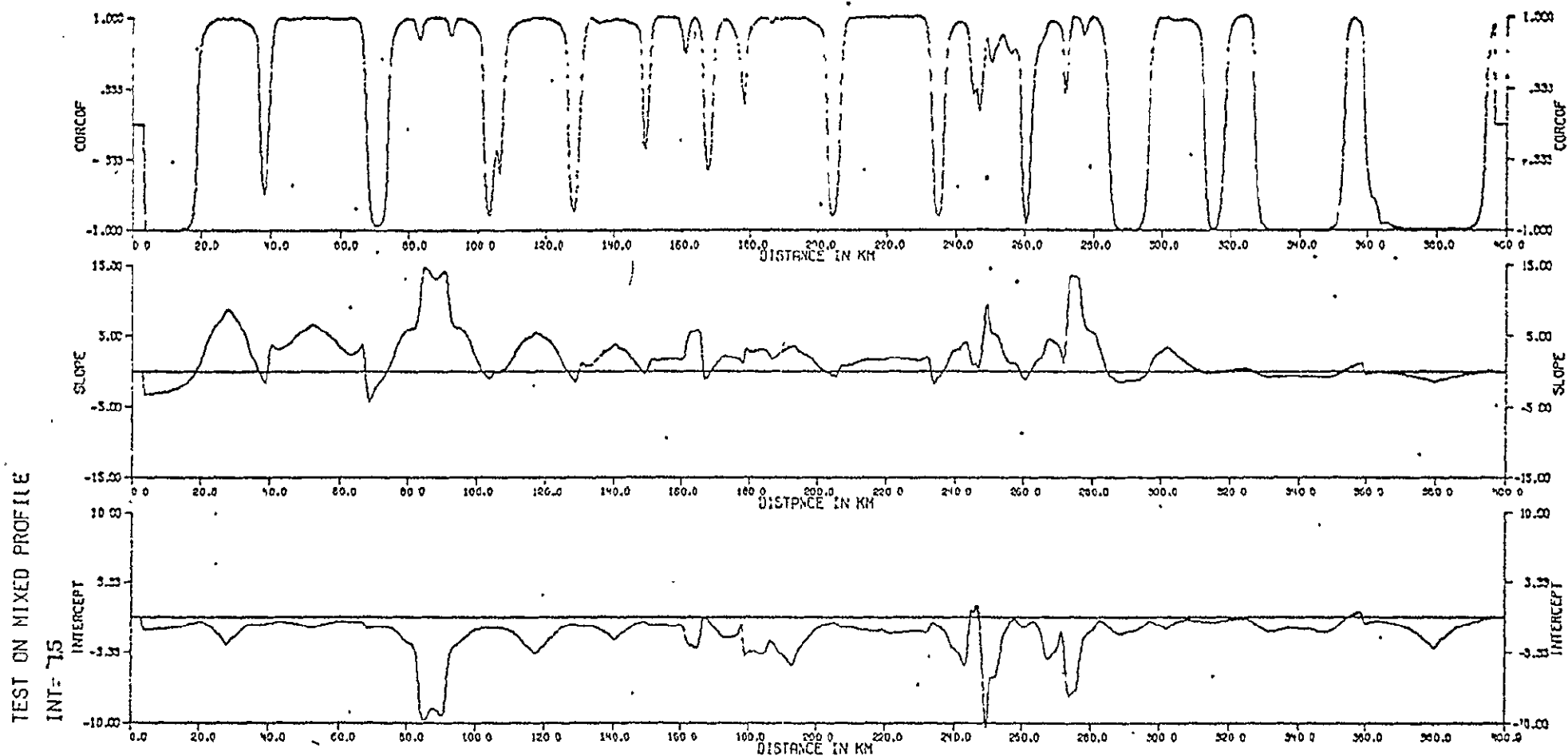


Figure 11. Internal correspondence analysis results for Profile 2 using a window of 7.5 km.

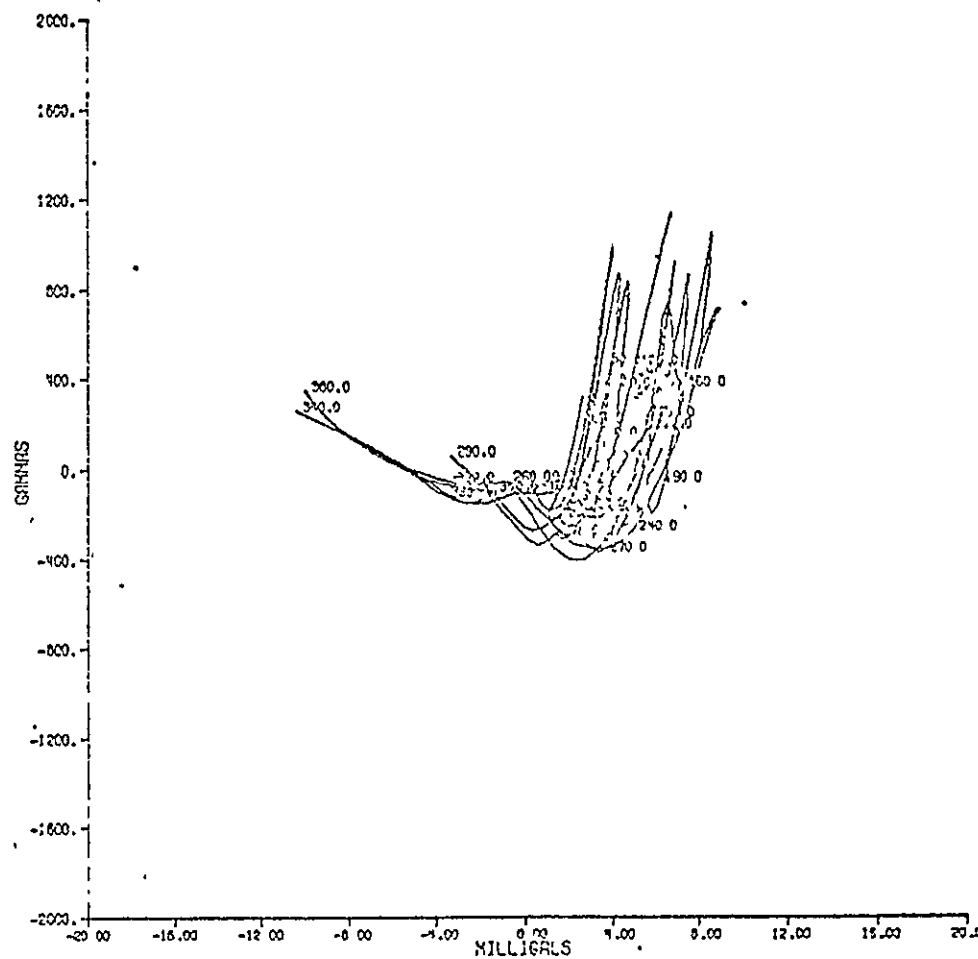
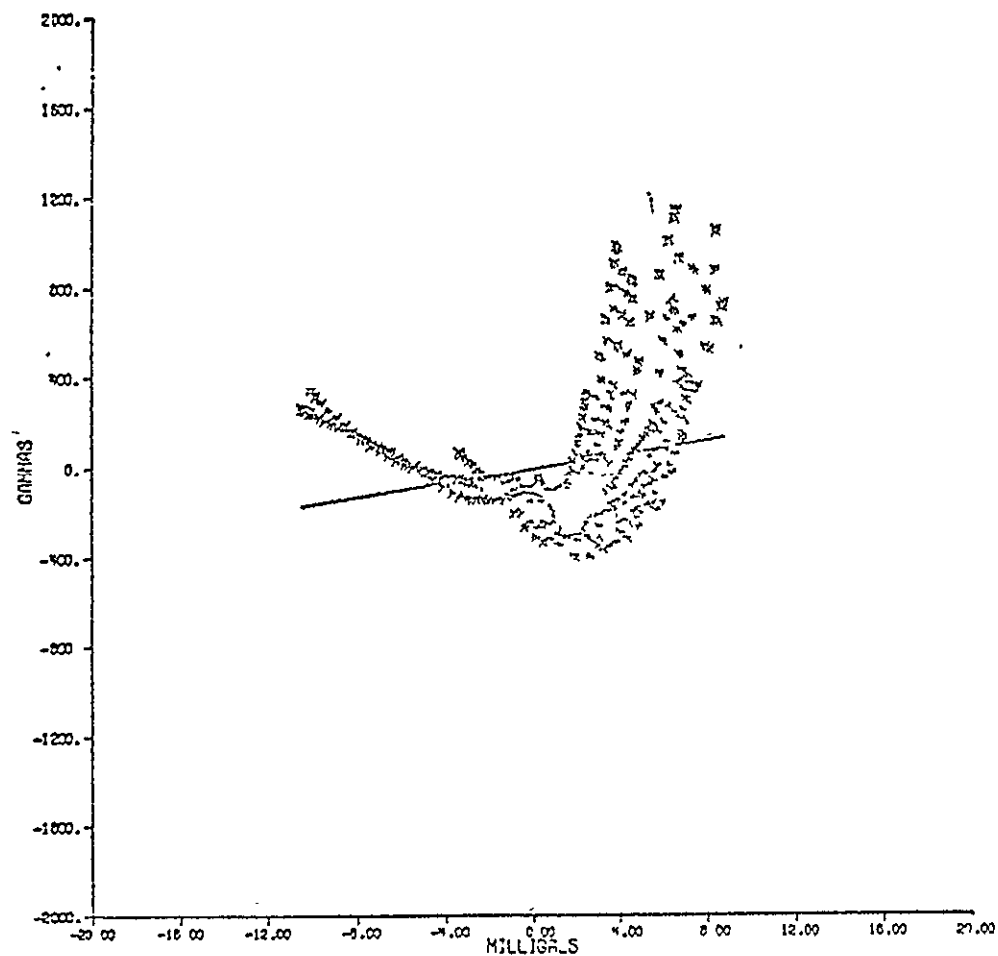


Figure 12. Scatter diagrams showing gravity vs. magnetic anomaly values for points along Profile 2.

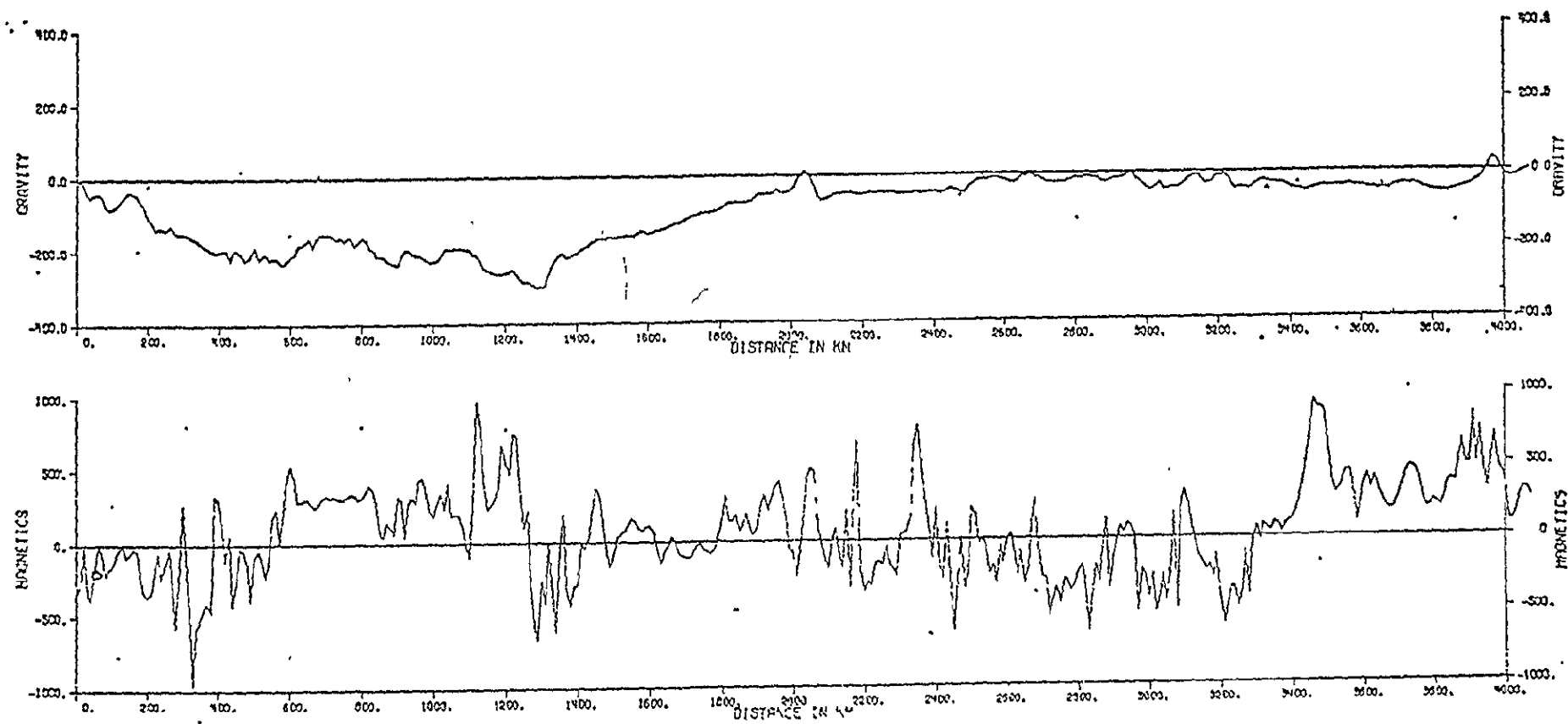


Figure 13. Woollard's (1943) transcontinental Bouguer gravity and vertical magnetic field anomaly profiles.

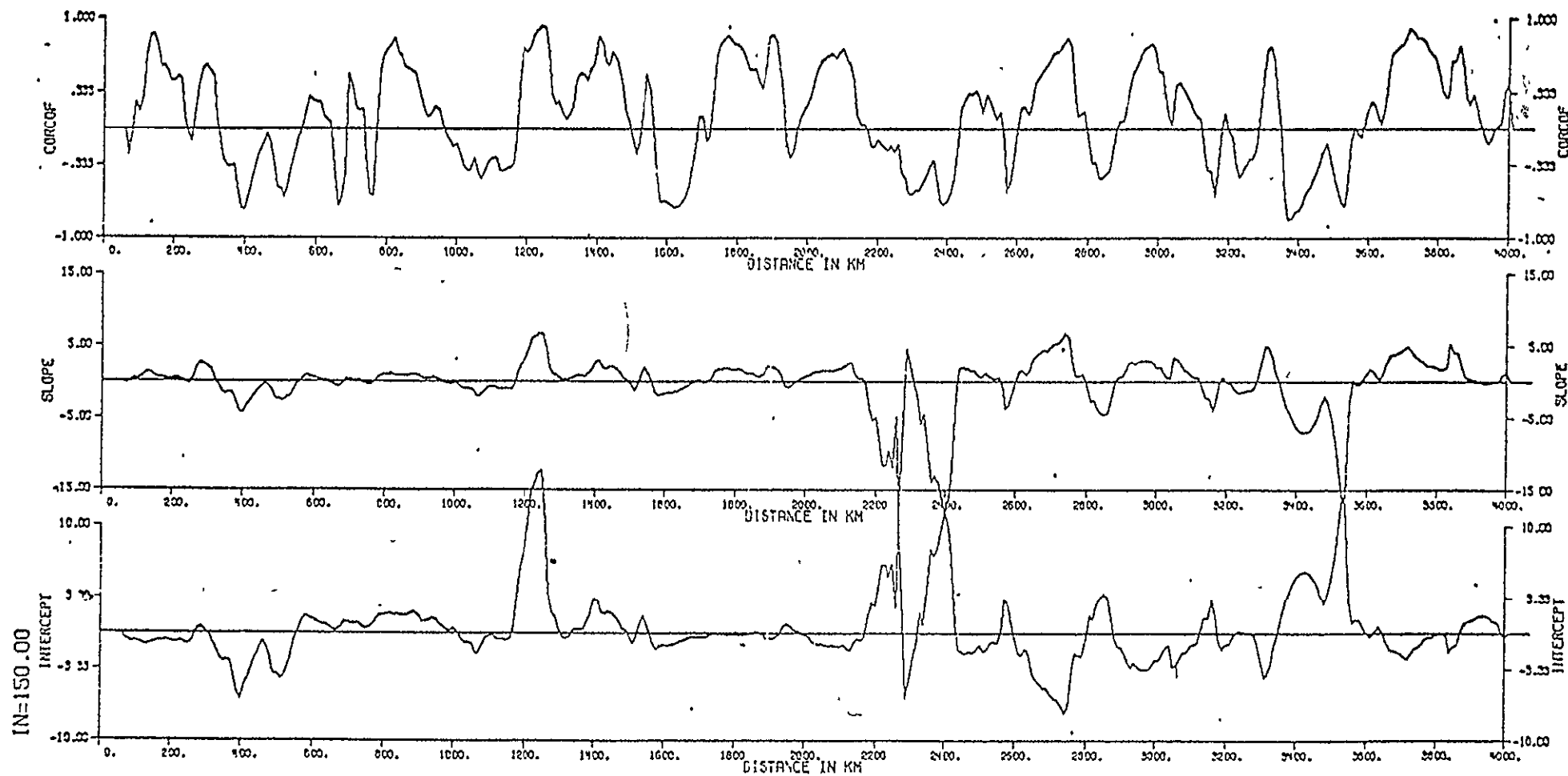


Figure 14. Internal correspondence results for Woollard's profile using a window of 150 km.

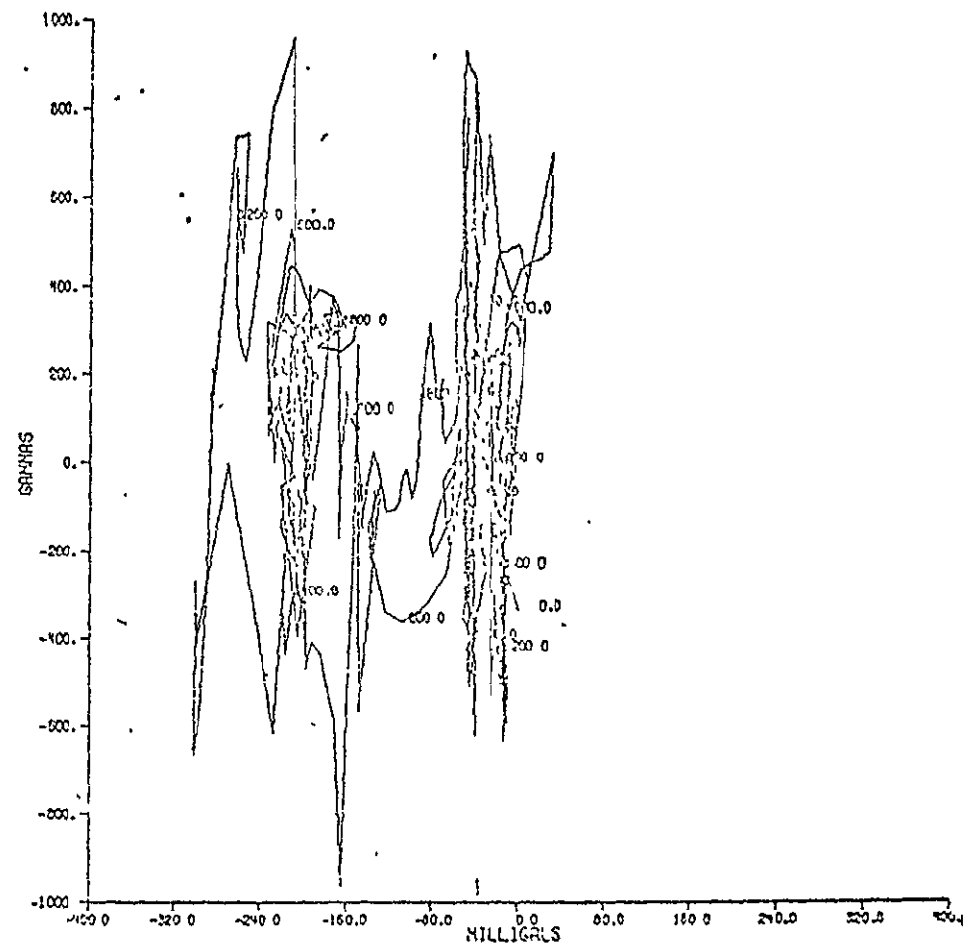
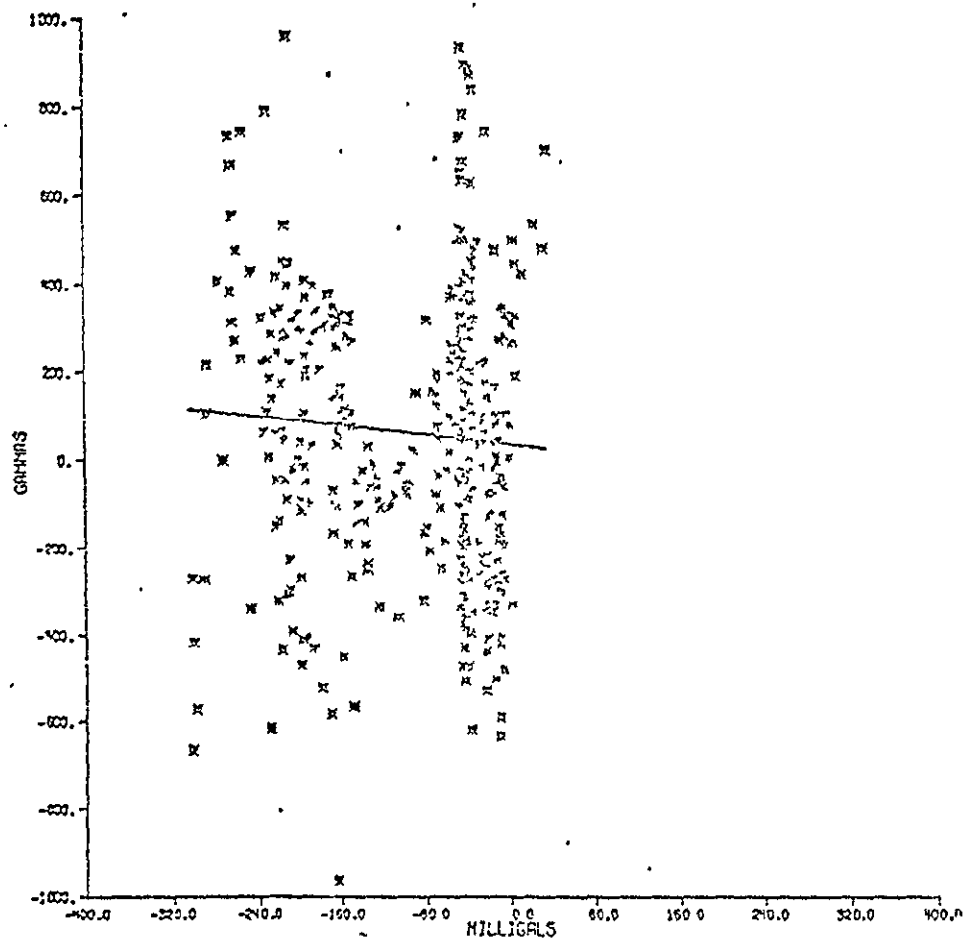


Figure 15. Scatter diagrams showing gravity vs. magnetic anomaly values along Woollard's profile.

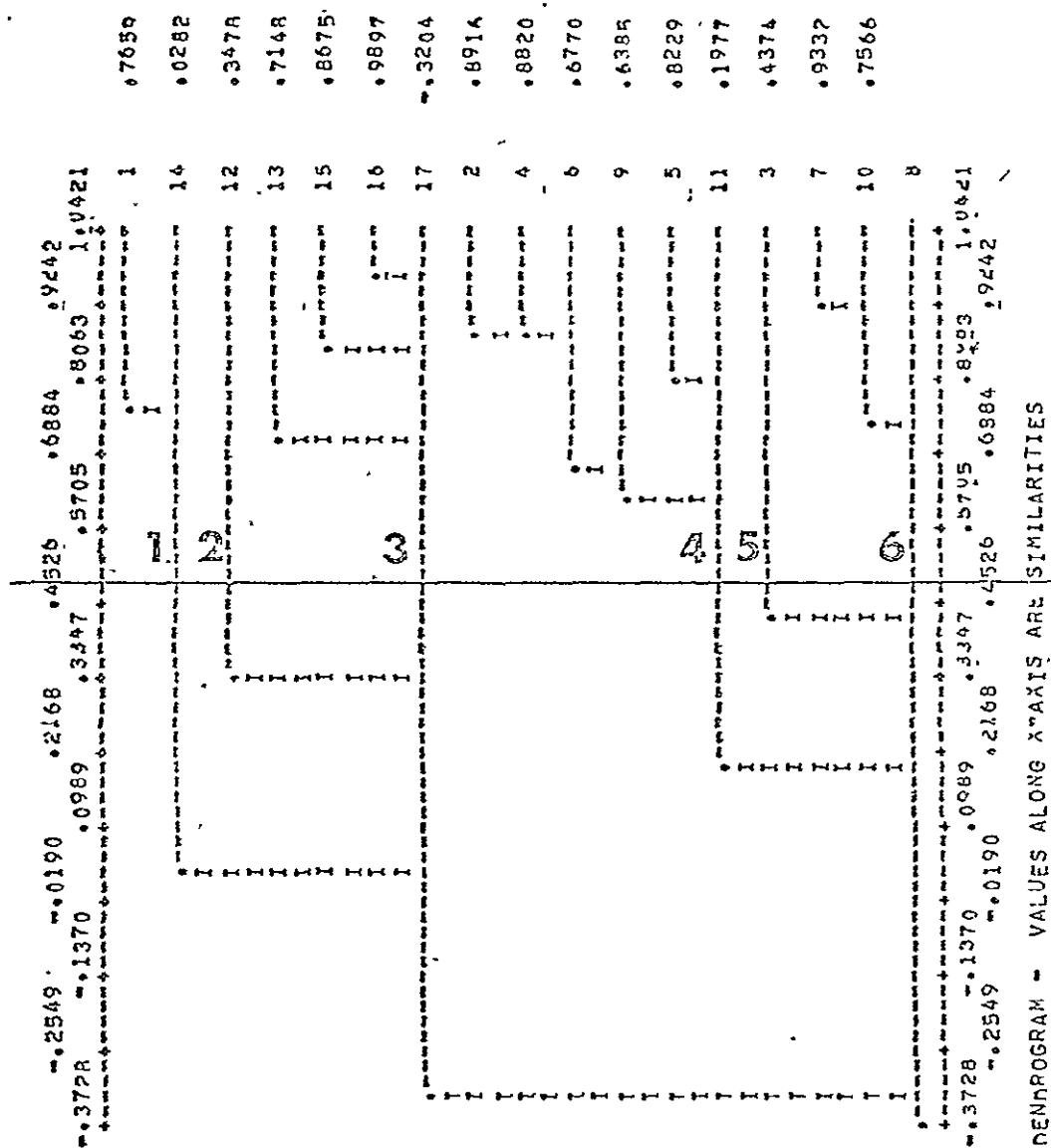


Figure 16. Cluster analysis dendrogram of Profile 2.

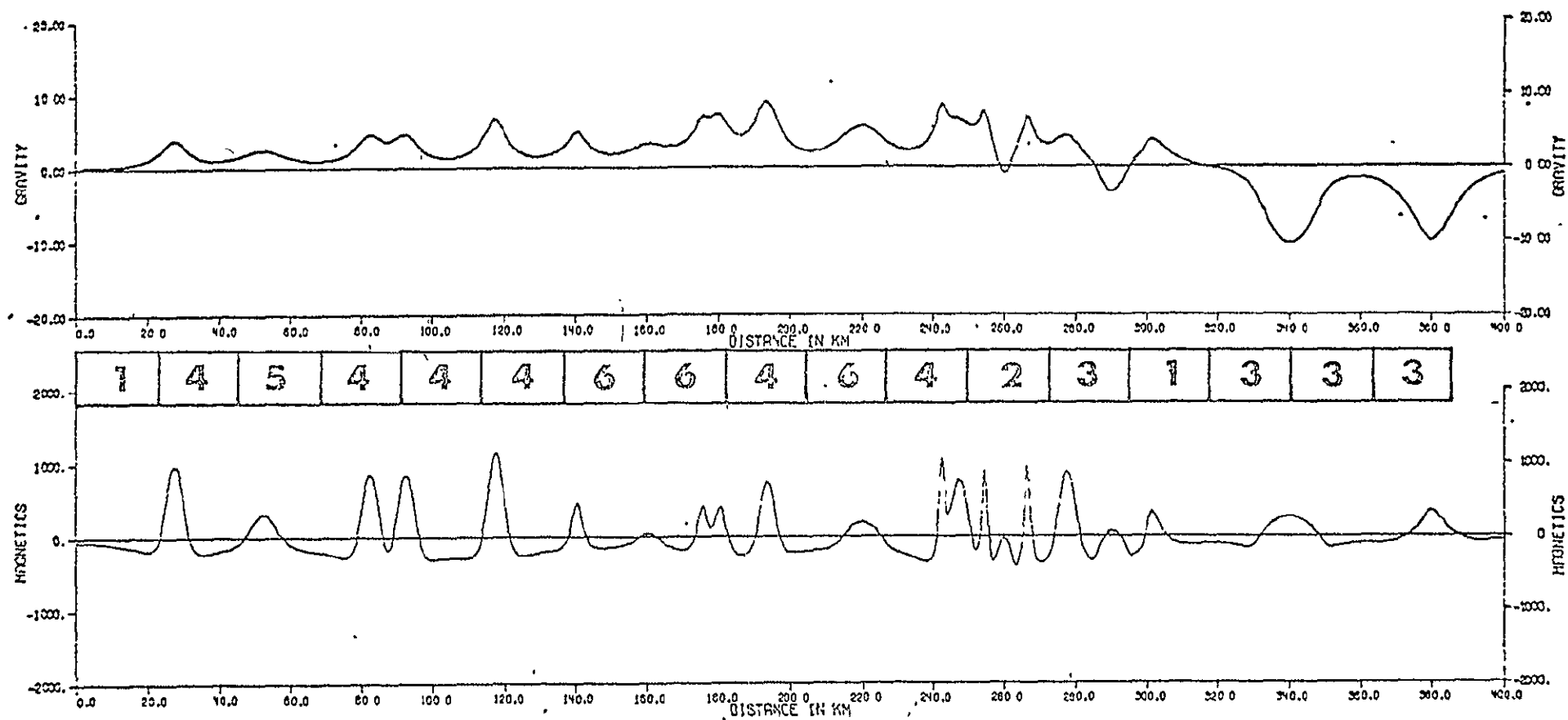


Figure 17. Results of cluster analysis of Profile 2 using a window of 22.5 km.

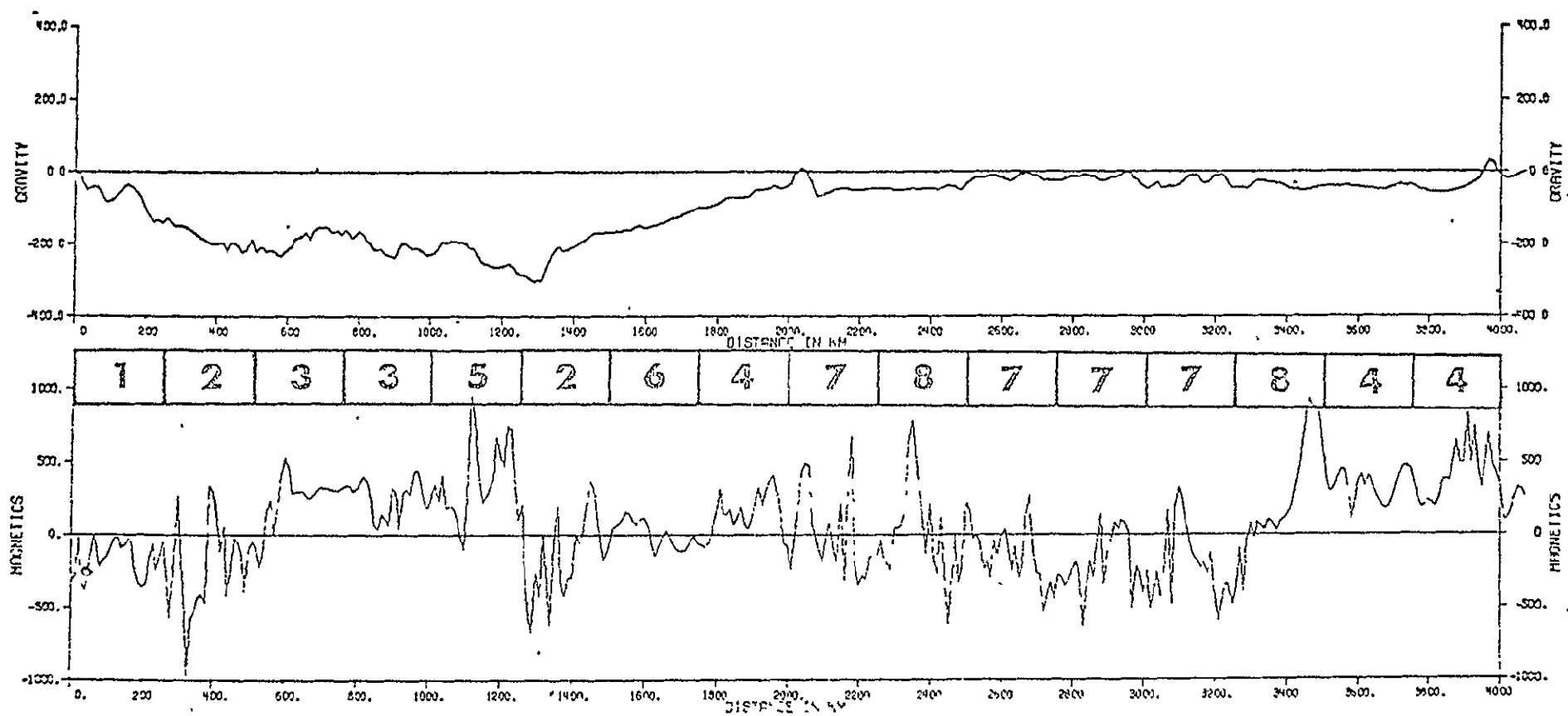


Figure 18. Results of cluster analysis of Woollard's profile using a window of 250 km.

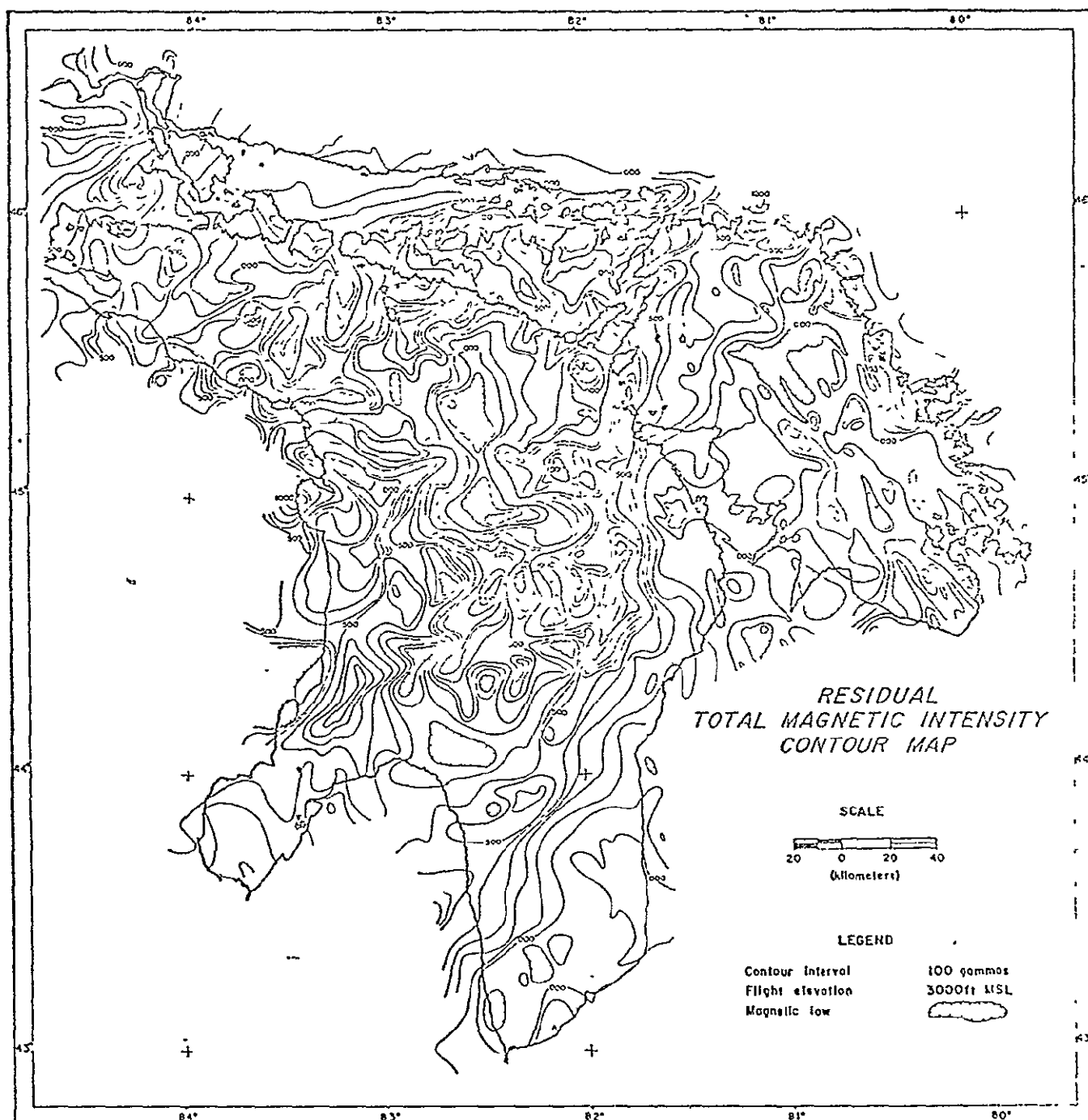


Figure 19. Residual total magnetic intensity map of Lake Huron (After O'Hara and Hinze, 1972).

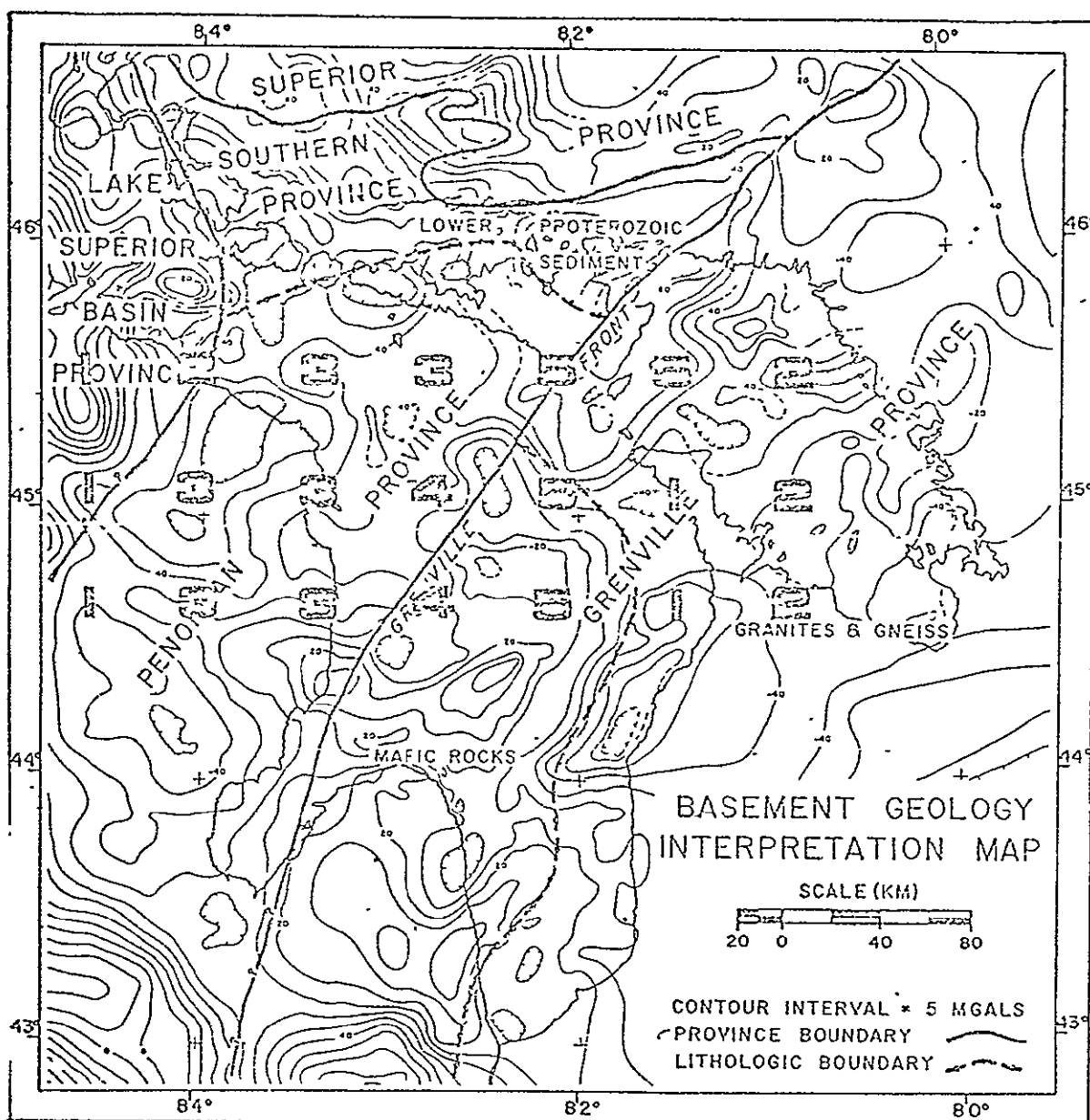


Figure 20. Bouguer gravity and basement geology interpretation map of Lake Huron showing results of cluster analysis.

APPENDIX

ANNOTATED BIBLIOGRAPHY OF
METHODS FOR CORRELATING MAGNETIC
AND GRAVITY ANOMALIES.

ANNOTATED BIBLIOGRAPHY OF
METHODS FOR CORRELATING MAGNETIC
AND GRAVITY ANOMALIES.

1. Bassett, K., 1972, Numerical methods for map analysis: Progress in Geography, v. 4, p. 217-254.

A comprehensive review of map analysis and correlating methods and their limitations are presented. A good reference list is included.

2. Bhattacharyya, B.K., 1967, Some general properties of potential fields in space and frequency domain: a review: Geoplotation, v. 5, p. 127-143.

A discussion of the properties of gravity and magnetic fields is presented. If a suitable combination of gravity and magnetic fields can be obtained it is possible to calculate the magnitude and direction of the magnetization vector.

3. Botezatu, R., and Colata, C., 1973, Cross correlation as an aid in simultaneous gravity and magnetic analysis: Geophysical Prospecting, v. 21, p. 472-483.

A non-normalized cross correlation function is used to relate gravity and magnetic intensities. Genetically related anomalies can be discriminated from anomalies produced by different geological bodies situated on the same vertical line.

4. Bott, M.H.P., Smith, R.A., and Stacey, R.A., 1966, Estimation of the direction of magnetization of a body causing a magnetic anomaly using a pseudo-gravity transformation: Geophysics, v. 31, p. 803-811.

The direction of magnetization of a two-dimensional body is estimated using an adaptation of Baranov's transformation of magnetic anomalies to pseudo-gravity anomalies. This method can be extended to three-dimensional bodies. Sources of error are briefly discussed. Both theoretical and observed examples are given.

5. Chereau, J.Y. and Naudy, H., 1967, Combined interpretation of gravity and magnetic data: U.N. ECAFE Mineral Resources Dev. Series No. 26, p. 464-477.

The importance of combined interpretation is stressed. Indirect comparison, direct comparison (using Poisson's theorem) and combined interpretation are described in a non-quantitative manner. Examples of various degrees of gravity and magnetic correlation are given.

6. Chorley, R.J., ed., 1972, Spatial Analysis in Geomorphology: Harper and Row, New York, 393 p.

This book includes several excellent articles on map analysis. Introductory article by Chorley contains very good list of references.

7. Corbett, J.D., Hinze, W.J. and Secor, G.B., 1967, A regional geophysical study of the Port Coldwell Complex, Ontario (abstr.): Inst. on Lake Superior Geology.

Combined gravity and magnetic analysis utilizing Poisson's theorem was employed to investigate the physical properties of the intrusive.

8. Cordell, L., and Taylor, P.T., 1971, Investigation of magnetization and density of a North American seamount using Poisson's theorem: Geophysics, v. 36, no. 5, p. 919-937.

A relationship is developed through Poisson's theorem between gravity and magnetic anomalies in the frequency domain for an isolated and uniformly magnetized body. A series of linear equations involving density, magnetization and calculated Fourier-series coefficients are used to solve for the three components of the total magnetization vector divided by the density. An example is given.

9. Curry, L., 1967, Quantitative geography: Canadian Geographer, v. 11, p. 265-279.

The author suggests fitting a polynomial to one surface and then reducing a second surface by the same expression; the measure of association is the proportion of the variance of the second map explained by the polynomial of the first.

10. Davis, John C., 1973, Statistics and Data Analysis in Geology: New York, John Wiley and Sons, Inc., 550 p.

This book discusses various correlation, clustering, and map comparison techniques. A good list of references is provided.

11. Eaton, G.P., Christiansen, R.L., Iyer, H.M., Pitt, A.M., Mabey, D.R., Blank, H.R., Jr., Zietz, I., and Gettings, M.E., 1975, Magma beneath Yellowstone National Park: Science, v. 188, p. 787-796.

Gravity and magnetic anomalies are correlated with geology and each other to decipher the subsurface geology of Yellowstone Park.

12. Garland, G.D., 1951, Combined analysis of gravity and magnetic anomalies: Geophysics, v. 16, p. 51-62.

A comprehensive discussion is given of the use of Poisson's theorem with particular application to gravity and vertical magnetic anomaly interpretation. An example of its use in physical property determination is given.

13. Garland, G.D., 1951, Comparisons of gravitational and magnetic anomalies over certain structures in Southeastern Ontario: The Canadian Mining and Metallurgical Transactions, v. 59, p. 340-345.

This article treats the general relationship between gravity anomalies and total field magnetic-anomalies utilizing Poisson's theorem. Examples of application are given.

14. Grossling, B.F., 1967, The internal magnetization of seamounts and its computer calculation: U.S. Geol. Surv., Prof. Paper 554-F, p. 26.

A method is presented for determining the magnetization of a body by an analysis of the magnetic anomaly in relation to the shape of the body. The method gives the total magnetization vector from a comparison of the observed field with three hypothetical fields obtained by assuming unit magnetizations in three orthogonal directions. A least-squares fit of a linear combination of the three fields to the observed one gives the magnetization components.

15. Hide, R., and Malin, S.R.C., 1970, Novel correlation between global features of the earth's gravitational and magnetic fields: *Nature*, v. 225, p. 605-609.

A correlation is made of gravity and magnetic fields of the earth's core. The coefficients of the spherical harmonic expansion of the magnetic field (with the secular variation removed) are used to correlate with gravity. They apply Student's t test and a stronger test by Brice to determine the significance of the correlation. Both show conclusive correlation. The geophysical significance of this correlation is reviewed.

16. Hinze, W.J., and Merritt, D.W., 1969, Basement rocks of the southern peninsula of Michigan; in *Studies of the Precambrian of the Michigan Basin*, ed. H.B. Stonehouse, Michigan Basin Geol. Soc., p. 28-59.

An interpretation of the basement rocks of the Michigan Basin in part by the correlation of gravity and magnetic anomalies is presented.

17. Henderson, John R., Jr., and Zietz, Isidore, 1958, Interpretation of an aeromagnetic survey of Indiana: *U.S. Geol. Surv. Prof. Paper 316-B*, p. 37.

Correlation between gravity and magnetic anomalies is discussed. The correlation is found to vary widely.

18. Kanasevich, E.R. and Agarwal, R.G., 1970, Analysis of combined gravity and magnetic fields in wave number domain: *J. Geophys. Res.*, v. 75, no. 29, p. 5702-5712.

Gravity and total field magnetic anomalies are correlated using two-dimensional fast Fourier analysis. They calculate J/σ ratio in the frequency domain using Poisson's theorem. Examples are given. Coherency test is used to measure the source correlation.

19. Karataev, G.I., 1964, Correlation scheme for linear prediction of crustal structure and composition from gravitational and magnetic anomalies: *Institute of Geology and Geophysics, Siberian Division, Academy of Science of the USSR, Novosibirsk. Geologiya i Geofizika*, no. 10, p. 33-49.

A description is given of a linear correlation model for gravity and magnetic features of the crust, together with the application of this model to prediction of geologic parameters.

20. King, T.R. and Zietz, I., 1971, Aeromagnetic study of the mid-continent gravity high of central United States: Bull. Geol. Soc. Amer., v. 82, p. 2187-2208.

An example of non-quantitative gravity and magnetic anomaly correlation and interpretation is given.

21. Leney, G.W., 1966, Field studies in iron ore geophysics: Mining Geophysics, v. 1, p. 391.

This paper gives an example of the use and the coincidence of magnetic and gravity anomalies with reference to a buried iron formation. An example is given.

22. Lidiak, Edward, 1971, Buried Precambrian rocks of South Dakota: Bull. Geol. Soc. Amer., v. 82, p. 1411-1420.

This is a survey article about the basement rocks of South Dakota. A comparison of the lithology from drill holes and the gravity and magnetic anomaly maps shows a strong correlation.

23. Lundbak, A., 1956, Combined analysis of gravimetric and magnetic anomalies and some paleomagnetic results: Geophysical Prospecting, v. 4, no. 3, p. 226-235.

A discussion is given on the application of Poisson's theorem to qualitatively determine the direction of remanent magnetization. Examples are given from Denmark and Northern Holland.

24. Lyubimov, A.A. and Lyubimov, G.A., 1968, The use of Poisson's ratio for geologic interpretation of gravity and magnetic anomaly analogs: Razvedka i Okhrana Nedr 34, no. 7, p. 38-42.

This article gives an example of the use of Poisson's theorem in an area (Kursk) where there is predominantly vertical magnetization and the source body is well isolated.

25. MacLaren, A.S. and Charbonneau, B.W., 1968, Characteristics of magnetic data over major subdivisions of the Canadian Shield: Geological Association of Canada, Proceedings, v. 19, p. 57-65.

Aeromagnetic (total field) data are presented for a large portion of Canada. The relationship of gravity and magnetic anomalies in the Canadian geological province is reviewed.

26. Mandelbaum, H., 1966, Comments on paper by D.F. Merriam and P.H.A. Sneath, Quantitative comparison of contour maps: J. of Geophys. Res., v. 71, no. 18, p. 4431-4432.

Critical comments are made on the three correlation techniques for comparison of maps discussed in Merriam and Sneath's paper (1966).

27. Merriam, D.F. and Lippert, R.H., 1966, Geological model studies using trend-surface analysis: J. of Geol., v. 74, no. 3, p. 344-357.

The authors have compared residuals from least squares maps and calculated the coefficient of association based on a number of matches of residual maps.

28. Merriam, D.F. and Sneath, Peter, 1966, Quantitative comparison of contour maps: J. of Geophys. Res., v. 71, no. 4, p. 1105-1115.

Trend surface analysis is used to generate meaningful characteristics of complex surfaces. Cluster analysis is applied to the data. Dendograms are generated which help characterize the map surfaces.

29. Meyer, Howard J., 1963, A combined magnetic and gravity analysis of the Sauble anomaly, Lake County, Michigan: Master of Science Thesis; Michigan State University, Michigan.

Poisson's theorem is used to assist in the interpretation of a local gravity and magnetic anomaly in Michigan.

30. Nettleton, L.L., 1942, Gravity and magnetic calculation: Geophysics, v. 7, no. 3, p. 293-310.

Methods for the evaluation of the potential field effects from various types of bodies is given. This article discusses the similarity of the gravity and magnetic formulas and the relationship of the amplitudes of gravity and magnetic anomalies.

31. Nettleton, L.L. and Elkins, T.A., 1944, Association of magnetic and density contrasts with igneous rock classifications: Geophysics, v. 9, no.1, p. 60-78.

An examination is made of the association of magnetite with rock densities in varying lithologies. Geologic implications of varying magnetization and density contrasts are discussed.

32. Newhart, Joseph A., 1975, Gravity and magnetic geophysical investigations of Sandusky, Seneca, and portions of Hancock and Wood Counties, Ohio: Master of Science Thesis; Bowling Green State University, Ohio.

Poisson's theorem is used to interpret a large gravity and magnetic anomaly in Ohio.

33. Oray, E., Hinze, W.J. and O'Hara, N., 1973, Gravity and magnetic evidence for the eastern termination of the Lake Superior syncline: Bull. Geol. Soc. Amer., v. 84, p. 2763-2780.

Visual spatial correlation of gravity and total magnetic intensity anomalies is presented and inverse modeling of both fields is conducted.

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38. Ross, H.P. and Lavin, P.M., 1966, In-situ determination of the remanent magnetic vector of two-dimensional tabular bodies: Geophysics, v. 31, no. 5, p. 949-962.

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39. Rudman, Albert and Blakely, Robert, 1965, A geophysical study of a basement anomaly in Indiana: Geophysics, v. 30, p. 740-761.

Example is given of the combined use of gravity with total vertical and horizontal magnetic intensity. Poisson's theorem was used to obtain information about the basement complex.

40. Rudman, Albert, Mead, J., Blakely, R. and Whaley, J.F., 1972, Precambrian geophysical provinces in Indiana: Proc. of the Indiana Acad. of Science, v. 81, p. 223-228.

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